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LONG TERM GROWTH OF A DEVELOPING ECONOMY

by

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Most growth models that have recently appeared are based on assumptions which are more relevant to the developed economies. Two important aspects of these economies are: 1) savings tend to approach a more or less stable rate[1], and (2) the productive structure tends to be settled on a more or less fixed technology[2]. If there have been any movements in the savings rate and capital output ratios of the developed countries, they have been downward. Under these conditions it is not surprising to find that in most of the models that have appeared, the rate of savings and the capital-output ratios have been assumed to be constant. And these assumptions will not be unjustified in the context of the developed economies with the characteristics mentioned above.

However, the above assumptions by their very nature, will be unjustified in the context of the developing economies. For two things are absolutely necessary for any economy to be labelled developing. Firstly capital should accumulate at an increasing rate, which in turn means that the savings rate should rise; secondly there must be, as far as possible, an optimal utilisation of resources, which, taken in conjunction with the first, means that the techniques of production and hence the capital-output ratio can be constant only by accident i.e., the techniques of production will keep changing. Due to these facts, the usual models of economic growth are not very useful for the developing economies.

In addition the developing economies have certain characteristics which make the usual models based on the customary multiplier and accelerator principles even less satisfactory. This is largely due to the following reasons. Firstly, almost all developing economies are short in capital. The

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(1) Cf. Capital Formation and Economic Growth, N.B.E.R., Princeton University Press 1955, P.6.

(2) Cf. P.S. ANDERSON, (The Apparent Decline in Capital Output Ratios) Quarterly Journal of Economics, November 1961.

scarcity of capital is the biggest impediment in the way of speedy development of these countries and froms the main bottleneck. The role of the accelerator is, thus, very much weakened, if not eliminated. For, real investment cannot react to changes in national income in the same manner in a capital-scarce economy as it can do in a developed economy. It cannot exceed the volume of (exante) savings, given the international movement of capital, due to the simple fact of nonavailability of additional capital. Similarly, it can hardly fall below the volume of (exante) savings in a developing economy owing to the existence of pent-up demand for consumption goods which ensures a ready market for goods produced and supplied.

In a dveloping economy, the consumption behaviour of the self-subsistent households particularly in the early stages of development may be significantly different from the rest of the spending units, in that an increase in the household output might not generate the same sequence of increased consumption and investment as it might do normally. This is due to the allegedly negative elasticity of work-effort to output. The influence of these households, however, seems to be limited in the developing economies or (perhaps more correctly) it is insulated from the general functioning of the economy. For the over-all effect in all the countries moving on the path of development has been a general rise in work-effort and consumption at an increasing rate. Moreover, the self-subsistent character of the households weakens as the economy grows, education spreads and the means of communication and transportation are developed. Nontheless, it is obvious that the existence of self-subsistent households does modify the multiplier effect by dampening it.

It should be pointed out, however, that the role of the multiplier has not been very dynamising even in the growth models pertaining to the developed countries, it is still less so in models pertaining to the developing countries. In fact it is very much subdued and its dynamising effect is generated through a process which is the reverse of that observed in the case of the developed countries, i.e. the lesser the rate of growth of consumption, the greater the rate of growth of investment and the greater the expansion.

After these preliminary remarks, we proceed to construct a long term model of economic growth for a developing economy. Let the initial amounts of labour and capital be L_o and K_o respectively. We assume that the labour force increases exogenously at a constant rate of per year. The rate of capital formation depends upon the rate of savings. We assume the savings rate at increase at a rate of s per year from an initial rate S_o. The total output in the economy in the t_{th} year is

$$P_t = F(L_t, K_t) \quad (1)$$

where L_t is the labour force in the t_{th} year and K_t the amount of capital. We have

$$L_t = L \; e^{\lambda t} \quad (2)$$

$$\dot{K}_t = S_o \, e^{st} \, P_t \quad (3)$$

We finally assume that the productive activity of the economy is adequately described by the Cobb-Douglas production function, so that

$$\dot{K}_t = S_o e^{st} L_t \, K_t$$

$$= S_o L_o^{\alpha} \, e^{(s+a\,)^t} \, K_t^{\beta} \quad (4)$$

the solution[1] of (4) is

$$K_t = \left\{ \frac{(S_o L_o^{\alpha}(1- \;)}{s+\alpha} \; e^{(s+\alpha\,)^t} + c \right\}^{\frac{1}{1-}} \quad (5)$$

where c is the integration constant which can be determined by initial conditions and is given in our case by

$$K_o = \frac{S_o L_o^{\alpha} (1- \;)}{s+\alpha} + c^{\frac{1}{1-}}$$

$$\text{or } c = K_o^{1-} - \frac{S_o L_o^{\alpha}(1- \;)}{S+\alpha}$$

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(1) Cf. E.L. INCE? Integration of Differential Equations, 1946, fourth edn. p. 22.

substitution of (2) and (5) in (1) completes the model. We can compute the future growth of national product for given values of S_0, L_0, a, , s, . We illustrate the model with figures that are more or less relevant to Indian conditions. As we are interested in percentage changes rather than in absolute values, we take $L_0 = 1 = K_0$, S_0 , = .1; take the usual values for α and β, i.e. 75 and .25; and stipulate that s = .005, and λ = .02.

For these values

$$K_t = (3.75e^{2t} - 2.75)^{3/4} \qquad (6)$$

From (2), (6) and (1), we compute the following table tracing the long-term growth of a developing economy.

TABLE L.
Showing the Long-Term Growth of National Production.

t	L_t	K_t	P_t	P_t/K_t	P_t/L_t
(1)	(2)	(3)	(4)	(5)	(6)
0	1	1	1	1	1
5	1.105	1.557	1.204	.77	1.0896
10	1.221	2.238	1.421	.63	1.1646
15	1.350	3.059	1.656	.54	1.2266
20	1.492	4.032	1.913	.47	1.2822
25	1.649	5.181	2.196	.42	1.3317

The above table shows that for the values of the parameters, the value of national product is doubled in 20 to 25 years (col.4). This conforms well to the targets fixed in the successive Indian Plans[1]. The Indian planners have been generally thinking in terms of doubling the

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(1) Cf. Third Five Year Plan, Govt. of India, p. 21.

national product in 25 years. The output-capital ratio (col.5) keeps falling at a diminishing rate. This again is in keeping with the underlying assumptions of the Indian Plans[1]. The national product per worker shows a much lower rate of increse[2]. It registers a rise of 33 per cent only in 25 years. This is at variance with the assertion of the first two Five Year Plans that the level of national income in 1950-51 could be doubled by 1970-71 and that of per capita income by 1977-78. The mistake was realised by the Third Five Year Plan, though no systematic attempt has been made to estimate the rate of per capita growth of income[4].

It is interesting to study the movement of the consumption level in the framework of the theoretical model we have constructed. The main concern of the people at large in a developing economy is the rise of the per capital consumption level, even though the growth of the national product is important.

It is obvious that the aggregate consumption is equal to the total net product minus the net investment, i.e.,

$$C_t = P_t - \dot{K}_t \qquad (7)$$

where C_t is the aggregate consumption in period t. As we have assumed that the initial rate of savings is .1, and our unit of measurement of labour and capital is such that $L_o = 1 = K_o$ and hence $P_o = 1$, we must have $C_o = .9$ (for $C_o = P_o - K_o = 1 - .1 = .9$). As $C_t = P_t . (1 - S_o e^{st})$, putting as before $= .02$. and $s = .005$, we have the rate of growth of per capita consumption.

$$w = \frac{\log P_t/C_o\,(1-S_o^{est})}{t} -$$

$$w = \frac{\log P_t/.9\,(1-.1\,e^{.005t})}{} - .02$$

1)Cf. Second Five Year Plan, P.11. Here the incremental capital-output ratio has been shown as increasing, i.e., the output-capital ratio is falling. Figures in Table 1 are not real, because of our abritrary units of measurement of initial values of labour and capital. They, however, purport to show the tend of the output-capital ratios.

2) If we assume a worker's family of five members, the per capita output will be one-fifth of the per worker output.

3) Cf. Third Five Year Plan, Govt. of India, p. 21.

The following table gives the values of w corresponding to 5, 10, 15, 20, 25 years.

TABLE 11.

Showing the Movement of Per Capita Consumption Level for the Assumed Values of Variables

t	Total product	Total consumption	Rate of growth of aggreate consumption per year over the last t years	Rate of growth per capita consumption per year over the last t years
(1)	(2)	(3)	(4)	(5)
0	1	9.	----	---
5	1.209	1.0805	.0365	.0166
10	1.421	1.2715	.0346	.0145
15	1.656	1.4775	.0331	.013
20	1.913	1.6922	.0316	.0118
25	2.196	1.9471	.0309	.0107

For the assumed initial values of the variables and assumed values of the prameters, table 11, col. 3, gives the magnitude of total consumption. The level of aggregate consumption in the 25th period is more than double of the level in the initial period as is that of the total product (col. 2). The aggregate consumption per annum increases at a simple rate of roughly around 4 per cent, but the compound rate reckoned from the initial rate decreases at a diminishing rate from3.6 per cent to 3.1 per cent roughly. The overall rate of growth of per capita consumption is 1.6 per cent for the first five years, it is 1.45 per cent for the first 10 periods but it goes on decreasing but at a diminishing rate.

Planners and policy-makers are more frequently concerned with the inverse of the problem stated above. In what has preceded, we assume a certain annual rate of increase in the rate of savings. But the equal (if not more) important problem is to ensure a certain given increase in the per capita consumption per annum. In this case the rate of savings or investment in each year will be dependent upon magnitudes of output and consumption in each year. In fact, as argued earlier, the volume of investment in each year will be equal to the difference between the volumes of output and consumption, i.e.,

$$\mathring{K}_t = P_t - C_t \qquad (8)$$

We further envisage that per capita consumption is to be increased by θ per cent. So that

$$C_t = C_o \, e^{\lambda + \theta}$$

where C_o is the initial level of consumption and λ is the rate of increase of population as before. Now

$$\mathring{K}_t = L_t^{\alpha} \, K_t^{\beta} - C_o \, e^{(\lambda + \theta)t} \qquad (9)$$

Further we have:

$$K_t = K_{t-1} + \mathring{K}_{t-1} \qquad (10)$$

(10) implies that savings of a period are invested in the succeeding period or else they are invested in the same period, but start to yield output in the succeeding period. It is obvious that if we know $\mathring{K}_t$'s for successive t's, we can find out K_t's for all t's, from (10). $\mathring{K}_t$'s can be determined by solving (9). A general solution of (9), however,

is not possible, we therefore adopt the pedestrian method[1] of finding K_t's for given initial values of L and K, and assumed values of λ and θ by the interative pocess. The following table gives the values of total output, consumption and investment for the first 10 years under the assumption that the per capita consumption increases at the rate of one per cent per annum so that $\theta = .01$. Further as the initial saving is supposed to be .1, $C_0 = .9$; the values of other variables and paramenters remaining the same as before.

TABLE III.
Showing Growth of National Production under the Assumption of an Annual Increase of one percent in per capita Consumption.

1	Total capital stock	Total output	Total investment	Total consumption	Capital output ratio	Rate of savings
(1)	(2)	(3)	(4)	(5)	(6)	(7)
0	1	1	.1	.9.	1	.1
1	1.1	1.04	.113	.927	1.057	.109
2	1.213	1.081	.125	.956	1.122	.116
3	1.338	1.125	.140	.985	1.189	.125
4	1.478	1.171	.154	1.017	1.262	.132
5	1.632	1.218	.172	1.046	1.339	.141
6	1.804	1.268	.191	1.077	1.422	.151
7	1.995	1.323	.212	1.111	1.508	.160
8	2.207	1.374	.230	1.144	1.606	.167
9	2.437	1.430	.251	1.179	1.704	.176
10	2.688	1.488	.273	1.215	1.808	.184

1-The adoption of the pedestrain method is not by choice. Equation (9) as it stands is not amenable to solution.

I tried a solution by linearising it through substituting f(t)K for $L^{\alpha} K_t^{\beta}$ where f(t) is a function giving output-capital ratios for t, so that (9) is transfromed into $\dot{K}_t = K_t . f(t) - C_o e^{(\lambda+\theta)t}$

The solution of this equation is (cf. L. INCE. op. cit., pp. 18-19.

$$K_t \, e^{\int -f(t)} + \int C_o \, e^{(\lambda+\theta)t} \, e^{\int -f(t)} = \text{some constant.}$$

Unfortunately any decreasing function for f(t) which we can assume makes the second term in the right-hand side of the above equation non-integrable.

For the solution of equation(9), I consulted Prof. EDMUND PLINNEY, University of California, besides the mathematicians of my University who kindly pointed out in details the diffculties involved in solving a non-linear mixed differential equation of the type of (9).

Comparing tables II and III we see that total consumption in latter is lower than that in the former for the first 10 periods (for which figures have been given in table III). This means that a mere half a per cent annual increase in the rate of savings from an initial rate of 10 per cent is compatible with a more than one per cent annual increase in the rate of per capita consumption per annum which is implied in table III. As column 5 of table II shows, this incremental rate is more than one per cent for all the years, though it is gradually diminishing. The latter, in turn, implies an obvious fact that a constant annual increase in the rate of savings under the type of models we have discussed brings about a diminishing rate of annual increase in the per capita consumption[1].

The simple model outlined above possesses the general defects of the growth models that have been developed in the recent past. These modles take savings rate, technology and preferences, as given and then set-up to work out a growth process mostly on the basis of accumulation of capital. The present model too assumes the rate of savings as given, but increasing at a constant rate. We have not gone behind the savings rate to study the forces that are at work in bringing savings rate to the levels assumed. The increases in savings rate may be either induced by deliberate monetary fiscal policies or they may be brought about quite endogenously due to increases in per capita incomes as the development process starts. In any complete model these forces will have to be thoroughly analysed and integrated.

1- After a few periods, rate of growth of per capita consumption in table II will become less than one per cent. As the rate of savings in table III increases at a faster rate than that in tables I and II, both the total output and consumption will be higher under the assumptions of table III after the critical period and thereafter keep increasing at a faster rate than that will be possible under the assumption of the first two tables.

Although the present model does not assume capital output ratio or the technology as given, yet it is based on a certain function i.e., the Cobb-Douglas. In an aggregative model, we do not really have any alternative choice[1]; but this cannot justify our approach as wholly sound. The limitations of the Cobb-Douglas function have been adequately discussed and a model based on a production function, will certainly partake of its weakness. The use of a production function, however, is an improvement over that of a capital output ratio, in that it allows for substitution of factors, choice or use of alternative technology commensurate with variations in the availability of resources. In the developing countries these two aspects are crucially important; hence the need for introducing a production function.

In an aggregative model of the type outlined here, the changes in the community's preferences can hardly be accounted for. The framers of such models implicitly assume that preferences and so relative prices remain unchanged. This is a major simplification in itself, but when we consider our failure to take into account the inter-temporal preferences with which the rate of savings is inalienably connected, the situation becomes grossly oversimplified [2]. It is, therefore, advisable that in the course of a study of aggregative growth models recently developed, and particularly the model presented here, these limitations are kept in mind.

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1) An alternative production function has been put forward by ARROW, CHENERY, MINHAS and SOLOW in the Review of Economics and Statistics, Aug. 1961. This function is more relevant in case of individual industries than in that of the whole economy. The function, however, is even less amenable to mathematical operations than the Cobb-Douglas.

2) The rate whith which future should be discounted has remained an intractable problem in social sciences. A very rough and crude approach, sometimes adopted by writers, is to relate it ot the prevailing interest rates. Any rate of interest, however, is a very poor indicator of the real inter-temporal preferences of the community.