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Statistical Models for Some Economic Phenomena

By

DR. M. W. Mahmoud

&

M. V. Radwan

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CAIRO
3 MOHAMED MAZHAR - ZAMALEK

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1 - Introduction

It is of great importance to know the theoritical model that fits a given frequency distribution for any economic phenomenon. This will help in taking decisions or drawing plans.

Two particular economic phenomena are discussed in this paper :-

(i) The distribution of employees according to their wages

(i i) The distribution of households according to their percentage of expenditure on food stuffs for rural and urban regions.

The method adopted in this study deals with fitting Pearsonian system of curves. This system is derived by solving the the differential equation

$$\frac{1}{y}\frac{dy}{dx} = -\frac{x+b}{a+bx+cx^2} \qquad (1)$$

where

$$a = \frac{4\beta_2 - 3\beta_1}{2(5\beta_2 - 6\beta_1 - 9)}\sigma^2$$

$$b = \frac{\sqrt{\beta_1}(\beta_2 + 3)}{2(5\beta_2 - 6\beta_1 - 9)}\sigma \qquad (2)$$

$$c = \frac{2\beta_2 - 3\beta_1 - 6}{2(5\beta_2 - 6\beta_1 - 9)}$$

The integration of (1) ends in a form which depends on the roots of $a + bx + cx^2 = 0$ i.e. on the quantity

$$K = b^2 / 4ac = \frac{\beta_1(\beta_2 + 3)^2}{4(2\beta_2 - 3\beta_1 - 6)(4\beta_2 - 3\beta_1)} \qquad (3)$$

where $\beta_1 = \mu_3^2 / \mu_2^3$ & $\beta_2 = \mu_4 / \mu_2^2$.

For example :

if K is < 0, we have type I which is of the form

$$y = y_o \left(1 + \frac{x}{a_1}\right)^{m1} \left(1 - \frac{x}{a_2}\right)^{m2} \qquad -a_1 \leq x \leq a_2$$

if K is > 1, we have type VI which is of the form

$$y = y_o (x - \alpha)^{q_2} x^{-q_1} \qquad \alpha \leq x \leq \infty$$

if 0 K 1 , we have type IV which is of the form

$$y = y_o e^{-\nu \tan^{-1} x/a} \left(1 + \frac{x^2}{a^2}\right)^{-m} \qquad -\infty < x < \infty$$

2- The distribution of employees according to their wages.

Pearsonian System of Curves is fitted to the following data:-

(i) The frequency distributions of employees according to their wages for the whole economy (except the agrarian sector) in the years 1956, 58, 59, 60, 61, 62, 66.

(i i) The frequency distributions for each economic activity (except the agrarian sector) in the year 1966.

The results are summerized in tables (1) & (2)

Table (1) The statistical indicators for the frequency distributions in the years 1956, 58, 59, 60, 61, 62 & 66

years	μ'_1	μ_2	β_1	β_2	K
1956	335.1	114.2×10^3	5.25	8.49	-2.00
58	331.3	$10;19 \times 10^3$	5.42	9.11	-2.44
59	331.2	103.2×10^3	5.47	9.12	-2.40
60	329.0	104.5×10^3	5.59	9.16	-2.34
61	327.5	103.5×10^3	5.72	9.31	-2.38
62	319.2	98.5×10^3	6.08	9.84	-2.61
66	374.3	108.4×10^3	5.20	8.53	-2.06

The values of K are all < 0, which indicates that all the frequency distributions have type I.

Table (2) The statistical indicators for the frequency distributions for the different economic activities in 1966.

Economic activities	μ'_1	μ_2	β_1	β_2	K
Mining & Qua.	450.1	172.9×10^3	2.94	5.02	-8.80
Manufacturing industries	324.4	77.1×10^3	7.26	11.87	-3.87
Construction & building	389.6	125.4×10^3	5.04	7.84	-1.68
Electricity -gas	513.4	123.1×10^3	2.78	5.67	-1.21
Trade	580.7	198.7×10^3	1.57	3.58	-0.50
Transportation	475.5	113.5×10^3	3.74	6.74	-1.76
Services	346.2	91.8×10^3	5.90	9.75	-2.68

The values of K are all < 0, which indicate that all the frequency distributions have type I.

3 - The distribution of households according to their percentage of expenditure on food stuffs

The data applied are published in the study of Family Budget by sampling 1958/59.

The results are summerized in table (3).

Table (3) Statistical indicators for the frequency distributions of households according to their percentage of expenditure.

Food Stuffs		μ'_1	μ_2	β_1	β_2	K
Cereals & Starches	U	1.78	1.64	3.33	7.82	-13.41
	R	3.08	2.67	1.62	4.53	-0.96
Dry beans	U	18.30	79.78	0.56	3.50	- 0.73
	R	31.78	95.28	0.30	2.85	- 0.20
Meat, Fish, Eggs	U	10.43	12.53	0.21	0.93	- 0.21
	R	11.16	10.23	0.13	3.61	+ 0.12
Oils & Fats	U	2.48	2.73	1.77	4.81	- 1.15
	R	1.83	1.74	2.73	7.01	-19.85
Milk & Milk Products	U	6.72	6.87	0.30	4.32	0.14
	R	8.31	17.64	1.28	5.51	1.09
Vegetables	U	4.54	3.59	0.19	2.14	- 0.07
	R	4.03	3.16	0.71	2.69	- 0.24
Fruits	U	2.96	2.91	1.37	4.05	- 0.70
	R	1.89	1.01	2.71	9.12	0.94
Sugar & Sugar food	U	3.96	3.38	0.96	3.52	- 0.49
	R	5.08	3.90	0.22	3.08	- 0.35
Other food Stuffs	U	3.93	13.23	11.40	21.25	14.35
	R	1.51	2.34	61.88	125.95	13.41

The values of K indicate that most of the frequency distributions have type I while few have either type VI or type IV.

4 - Comments

(a) Regarding the frequency distributions of employees according to their weekly wages, we find that all follow type I.

Economists believe that the Lognormal and the Pareto distributions are the best to graduate income distributions. The Pareto distribution is a special form of Type VI The Lognormal has values for β_1 & β_2 which lie in Type VI.

It is known that type I and type VI are orginally derived from the differential equation (1) where a, b, c are non-zeros and the quantity $a + bx + cx^2$ has two real roots having either different signs or the same sign.

(b) Regarding the frequency distributions of households according to the percentage of expenditure on food stuffs, we find that they are either of type I or type VI of type IV. These are originally derived from the same differential equation (1).

The common characteristic for these types lies in the fact that a, b, c are non-zeros. The only difference is that $a + bx + cx^2$ has two real roots with different signs for type I with the same sign for type VI but has imaginary roots for type IV.