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LIFE OF CAPITAL AND ECONOMIC GROWTH

or

PROCESSES OF FACTOR COMBINATION IN ECONOMIC GROWTH

by

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PROCESSES OF FACTOR COMBINATION IN ECONOMIC GROWTH*

1. Over-All Factor Substritution

So far two procedures of factor combination have been utilised in aggregative growth models that allow for continuous possibilities of factor substitution. One of them rests on the neo-classical approach which has been adopted by Tinbergen (1), Haavelmo (2), Solow (3), and Valavanis (4). This procedure assumes 'explicitly expressed possibilities of substitution between the <u>total</u> amounts of factors available', that is, as the total amounts of factory grow, they are simultaneously combined in their entirety with each other. Let there be only two factors of production, labour and capital, and their total amounts in a period t be denoted by $\bar{L}_t$ and $\bar{K}_t$, then the total output in period t will be

$$\bar{P}_t = F(\bar{L}_t, \bar{K}_t),$$

where F is the production function. The marginal productivity of the factors will be

$$\bar{w}_t = \frac{\partial F(\bar{L}_t, \bar{K}_t)}{\partial \bar{L}_t}, \qquad \bar{r}_t = \frac{\partial F(\bar{L}_t, \bar{K}_t)}{\partial \bar{K}_t}$$

We shall call $\bar{w}$ and $\bar{r}$ over-all marginal productivity of labour and capital respectively in view of the fact that they are derived through a procedure involving combination of total labour and total capital. If we assume that F is homogeneous of degree one, then[1)]

$$\bar{P}_t = \bar{L}_t \cdot \bar{w}_t + \bar{K}_t \cdot \bar{r}_t.$$

Further if we suppose that the elasticity of substitution of factors is unity, we have

$$\frac{\bar{L}_t}{\bar{K}_t} = A\,\frac{\bar{r}_t}{\bar{w}_t}$$

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where A is a constant depending on the form of F.[1] If we suppose the elasticity of substitution of factors, $\sigma \neq 1$, but equal to some constant we get

$$\frac{\bar{L}_t}{\bar{K}_t} = A \left(\frac{\bar{r}_t}{\bar{w}_t}\right)^{\sigma}.$$

The above two equations give the allocation of factors, given the elasticity of substitution.

2. Incremental Factor Substitution

However, this procedure of factor combination is feasible only when (1) the life of capital built is one production period only, or (2) the factors of production have been growing at the same rate for a very long time so that the ratio of their total amounts equals the ratio of their incrementals. The first of these is contradicted by the very definition of capital and the second has seldom held true in reality. This procedure of factor combination is, therefore, not feasible. It may be 'justifiably' resorted to when one is interested in studying a secular long term growth.

The second procedure of factor combination is that which has been adopted by Johansen (5) and Massel (6). The basic assumption underlying this approach is that 'there are substitution possibilities exante, but not ex-post', i.e., 'any gross increment in the rate of production can be obtained by different combinations of in capital and labour inputs.' The procedure followed by these writers combines gross investment in each period with the uncommitted labour supply in that period, thus ensuring full employment of both the factors. We shall call gross investment and uncommitted supplies of labour, fresh supplies of labour and capital and denote them $\tilde{L}$ and $\tilde{K}$, respectively. The (gross) increment of output in a period would accordingly be,

$$\tilde{P}_t = F(\tilde{L}_t, \tilde{K}_t),$$

1) In this paper we shall mainly deal with a production function with an elasticity of substitution equal to unity. In case when F assumes the form of the Cobb-Douglas production function $x = L^{\alpha} K^{\beta}$, then $A = \frac{\alpha}{\beta}$, or A = 3 if $\alpha = .75$.

and the resulting marginal productivity of factors

$$\tilde{w}_t = \frac{\partial F(\tilde{L}_t, \tilde{K}_t)}{\partial \tilde{L}_t} .$$

$$\tilde{r}_t = \frac{\partial F(\tilde{L}_t, \tilde{K}_t)}{\partial \tilde{K}_t} .$$

We call $\tilde{w}$ and $\tilde{r}$ incremental marginal productivity (or IMP for short) of labour and capital, respectively. If F is supposed to be homogeneous of degree one and the amounts of fresh supplies of labour and capital, $\tilde{L}_t$ and $\tilde{K}_t$, are combined according to IMP procedure, the resulting output of these factors in each period during the life-time of capital built will be

$$\tilde{P}_t = \tilde{L}_t \tilde{w}_t + \tilde{K}_t \tilde{r}_t .$$

If the capital built lasts θ periods, the total output resulting from $\tilde{L}_t$ and $\tilde{K}_t$ over θ periods will be $\theta\tilde{P}_t$.

Further if we suppose that the elasticity of substitution of factors is unity, we have

$$\frac{\tilde{L}_t}{\tilde{K}_t} = A \frac{\tilde{r}_t}{\tilde{w}_t} .$$

In case when $\sigma \neq 1$, but equal to a constant we have

$$\frac{\tilde{L}_t}{\tilde{K}_t} = A \left(\frac{\tilde{r}_t}{\tilde{w}_t}\right)^{\sigma} .$$

It is obvious that the second procedure is feasible, but it is contended here that it may be <u>inefficient</u>. This is stated on the basis of the fact that a third procedure of factor combination exists which is feasible and which may lead to higher levels of output from the same amounts of inputs.

3. Cumulative Incremental Factor Substitution

The basic idea behind the procedure to be suggested is that the combination of fresh supplies of factors in a project should not reflect the relative scarcity of factors in the current period only, but also the relative scarcities of their accumulated amounts in the succeeding periods till the last period of the life of capital built for the project. This procedure is feasible as Johansen's is, because it is possible to decide any combination of factors ex-ante, i.e., before the capital has been built; but in this case, the concern will be not to attain full employment of fresh (or unutilised) supplies of labour and capital by combining their total (fresh) amounts together, but to find that combination of these factors which reflects their relative scarcities in the successive periods as their fresh supplies grow during the life of the project. As it is to be expected, such a procedure will lead to a higher level of output. The point is that for the purpose of deciding the combination of fresh supplies of the factor in the current period, we will treat the fresh supplies of labour and capital forthcoming in the succeeding periods as perfectly 'fluid' and 'mobile', i.e., amenable to be used in alternative uses. Such a treatment is feasible for the same reason as the Johansen's procedure. If we can anticipate the amounts of fresh supplies of factors that will be available in the succeeding periods, we can choose a combination of the factors, that does not take into account their fresh supplies in the current period only, but also the supplies in succeeding periods as they get accumulated. The procedure of doing so is based on what we call cumulative marginal productivity of factors.[1] Let us suppose that we have to choose a combination of factors in period 1, and that the life of capital to be built is θ periods, the cumulative marginal productivity of labour and capital in period n ($1 \leq n \leq \theta$) is defined as

$$w_n^{*} = \frac{\partial F(L_n^{*}, K_n^{*})}{\partial L_n^{*}} \qquad r_n^{*} = \frac{\partial F(L_n^{*}, K_n^{*})}{\partial K_n^{*}}$$

where $L_n^{*} = \sum_{t=1}^{n} \tilde{L}_t; \quad K_n^{*} = \sum_{t=1}^{n} \tilde{K}_t.$

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1) It should be more appropriately called cumulative incremental marginal productivity in view of the fact that we cumulate only the incremental supplies of factors for their derivation, but to avoid too lengthy an expression we express it as in the text and write it CMP for short.

As before, the proportion in which the factors can be combined according to CMP's is given by

$$\frac{L_n^{**}}{K_n^{**}} = A\,\frac{r_n^{**}}{w_n^{**}} \qquad \text{when } \sigma = 1,$$

and $$\frac{L_n^{**}}{K_n^{**}} = A\left(\frac{r_n^{**}}{w_n^{**}}\right)^{\sigma} \qquad \text{when } \sigma \neq 1.$$

There is one difficulty, however, in the present case. In case of over-all marginal productivities or that of incremental marginal productivities, there is only one ratio of MP's to cope with, and hence the ratio of factors to be combined can be obtained straight away. This is not so with the CMP's, as they differ from period to period over the life-time of the plant to be installed in the initial period. Obviously some sort of an average of the varying CMP's will have to be considered. Out of the two types of averages, simple and weighted, the former seems to be logically more plausible, in view of the fact that the CMP's do imply weighting in respect of the amounts of factors that get accumulated. Hence a breakthrough can be made by using simple averages. We denote

$$w_1^{\theta} = \frac{1}{\theta}\sum_{n=1}^{\theta} w_n^{**}; \qquad r_1^{\theta} = \frac{1}{\theta}\sum_{n=1}^{\theta} r_n^{**},$$

where w_1^{θ} and r_1^{θ} represent the average of CMP's of labour and capital with respect to a plant going to be started in period 1 and to last till period θ. We can now find the ratio of factors that will be compatible with w_1^{θ} and r_1^{θ}, as before

(1) $$\begin{cases} \dfrac{L_1^{\theta}}{K_1^{\theta}} = A\,\dfrac{r_1^{\theta}}{w_1^{\theta}}, & \text{when } \sigma = 1, \\[2ex] \dfrac{L_1^{\theta}}{K_1^{\theta}} = A\left(\dfrac{r_1^{\theta}}{w_1^{\theta}}\right)^{\sigma} & \text{when } \sigma \neq 1. \end{cases}$$

If in a new plant labour and capital are combined in amounts L_1^{θ} and K_1^{θ}, which are in proportion given by the criterion developed above, the total output in each period will be under our assumptions of F being homogeneous of degree one,

$$P_1^{\theta} = L_1^{\theta}\, w_1^{\theta} + K_1^{\theta}\, r_1^{\theta}$$

and the total output over the life-time of the plant

$$OP_1^{\theta} = \theta(L_1^{\theta}\, w_1^{\theta} + K_1^{\theta}\, r_1^{\theta}).$$

4. A Numerical Example

The ideas stated above can be explained by a simple numerical example. Let the fresh amounts of labour and capital to be available in successive periods be

t	1	2	3	....	t
$\tilde{L}$	1	1	1	....	1
$\tilde{K}$	1	1.05	1.10	....	1+(.05)(t-1)

Let F be specified as $P = L^{\alpha} K^{\beta}$, $\alpha = .75$, $\beta = 1 - \alpha$, and let the fixed life of capital built be 2 periods only. Then using (1) of the previous section we have

$$(2) \qquad \frac{L_1^2}{K_1^2} = A \frac{\dfrac{\partial(\tilde{L}_1^{\alpha}\, \tilde{K}_1^{\beta})}{\partial \tilde{K}_1} + \dfrac{\partial(\tilde{L}_1 + \tilde{L}_2)^{\alpha} \cdot (\tilde{K}_1 + \tilde{K}_2)^{\beta}}{\partial(\tilde{K}_1 + \tilde{K}_2)}}{\dfrac{\partial(\tilde{L}_1^{\alpha}\, \tilde{K}_1^{\beta})}{\partial \tilde{L}_1} + \dfrac{\partial(\tilde{L}_1 + \tilde{L}_2)^{\alpha}\, (\tilde{K}_1 + \tilde{K}_2)^{\beta}}{\partial(\tilde{L}_1 + \tilde{L}_2)}} = .987767.$$

This shows that $\tilde{L}_1$ cannot be fully utilised, and the left-over of labour from period 1 is

$$\tilde{L}_1 - L_1^2 = 1 - .987767$$
$$= .012233$$

The labour available in period 2 is

$$\tilde{\tilde{L}}_2 = \tilde{L}_2 + \tilde{L}_1 - L_1^2$$
$$= 1.012233.$$

Using the same procedure as given by (2), again we have

$$\frac{L_2^3}{K_2^3} = .94992$$

$$L_2^3 = 1.05 \times .94992$$
$$= .99742$$

$$\therefore \tilde{\tilde{L}}_2 - L_2^3 = .00258$$
$$\therefore \tilde{\tilde{L}}_3 = \tilde{L}_3 + \tilde{\tilde{L}}_2 - L_2^3$$
$$= 1.00258$$

Repeated trials will give the proportion in which labour and capital should be combined in successive periods. To see that the method suggested above gives better results than that in case of IMP procedure even after the first round we compute the corresponding outputs. In the latter case the outputs will be

$$\tilde{P}_1 = \tilde{L}_1^{\alpha} \tilde{K}_1^{\beta}$$
$$= 1 \qquad \text{for } \tilde{L}_1 = 1 = \tilde{K}_1, \quad \alpha = .75, \quad \beta = 1 - \alpha,$$
$$\tilde{P}_2 = 1.01227 \text{ for } \tilde{L}_2 = 1, \ \tilde{K}_2 = 1.05, \ \alpha = .75, \ \beta = 1 - \alpha,$$

As the plants created last 2 periods, the total output from the fresh supplies of labour and capital in periods 1 and 2 is

$$2\tilde{P}_1 + 2\tilde{P}_2 = 4.02454.$$

The amounts of labour and capital utilised in period 1, according to CMP method outlined here are .999163 and 1 only. We assume, for the sake of comtogether is combined with the amount of fresh capital available in period 2, so that

$$P^{*}_{1} = {L^{2}_{1}}^{\alpha} \; {K^{2}_{1}}^{\beta}$$

$$= .990811 \quad \text{for } L^{2}_{1} = .987767, \text{ and } K^{2}_{1} = 1, \quad \alpha = .75, \quad \beta = 1 - \alpha.$$

$$P^{*}_{2} = 1.021512 \quad \text{for } \tilde{\tilde{L}}_{2} = 1.012233 \text{ and } \tilde{K}_{2} = 1.05.$$

The total output according to the CMP method is, then,

$$2P^{*}_{1} + 2P^{*}_{2} = 4.024646.$$

The output is greater in case of allocation according to the CMP procedure even when it is applied in the first round only. The difference is small but this is due to the small values we have taken to illustrate the idea and the small number of periods for which the capital lasts i.e. 2 only.

We can also make sure of the reverse case when $\tilde{K}$'s remain constant and L's increase, so that

t	1	2	....	t
$\tilde{L}$	1	1.05	....	$1+(.05)(t-1)$
$\tilde{K}$	1	1	....	1

Then

$$\frac{L^{2}_{1}}{K^{2}_{1}} = 1.01246$$

$$\therefore \quad K^{2}_{1} = .987692$$

$$\therefore \quad \tilde{\tilde{K}}_{2} = \tilde{K}_{2} + \tilde{K}_{1} - K^{2}_{1} = 1.012308$$

Therefore as in the preceding case

$$2\tilde{P}_{1} + 2\tilde{P}_{2} = 2(1 + 1.037270) = 4.074540$$

and

$$2P_1^{**} + 2P_2^{**} = 2(.996909 + 1.003063 \times 1.037270)$$
$$= 4.074712$$

Once again $2(P_1^{**} + P_2^{**}) > 2(\tilde{P}_1 + \tilde{P}_2)$ i.e. allocation according to the CMP procedure leads to a higher level of output even when we use it in one round only, i.e. in the first period.

5. Symbolic and General Proof

We can now proceed to give a symbolic proof of the numerical illustrations given above, still keeping the life of capital equal to 2 periods.

Let $\tilde{L}_2 = \tilde{L}_1 e^{\lambda}$ and $\tilde{K}_2 = \tilde{K}_1 e^{k}$ where $\lambda \gtrless k$

using (2) we have, when $\alpha + \beta = 1$ and $A = \frac{\alpha}{\beta}$

$$\frac{L_1^2}{K_1^2} = \frac{\tilde{L}_1}{\tilde{K}_1}(z) \tag{3}$$

$$\text{where } z = \frac{1 + \left(\frac{1+e^{\lambda}}{1+e^{k}}\right)^{\alpha}}{1 + \left(\frac{1+e^{k}}{1+e^{\lambda}}\right)^{\beta}}$$

In (3) if $\lambda < k$, then $z < 1$, and so in period 1, $\tilde{L}_1$ cannot be fully utilised, but $\tilde{K}_1$ can be fully utilised, so $K_1^2 = \tilde{K}_1$

$$\therefore L_1^2 = \tilde{L}_1 \times \frac{K_1^2}{\tilde{K}_1} z$$

$$= \tilde{L}_1 z$$

$$\therefore \tilde{\tilde{L}}_2 = \tilde{L}_2 + \tilde{L}_1 - L_1^2$$
$$= \tilde{L}_1(1 + e^{\lambda} + 1 - z)$$

Therefore the total output that can be produced with the fresh supplies of labour and capital available in periods 1 and 2 according to CMP approach using it only in period 1 as in the numerical example is

$$(4) \qquad 2(P_1^* + P_2^*) = 2\left[\tilde{L}_1^{\alpha} z^{\alpha} \tilde{K}_1^{\beta} + \tilde{L}_1^{\alpha}(1+e^{\lambda} + 1-z)^{\alpha} K_2^{\beta}\right]$$

and according to the IMP approach is

$$(5) \qquad 2(\tilde{P}_1 + \tilde{P}_2) = 2\left[\tilde{L}_1^{\alpha} \tilde{K}_1^{\beta} + \tilde{L}_2^{\alpha} \tilde{K}_2^{\beta}\right]$$

The difference between (4) and (5) is

$$(6) \qquad 2(P_1^*+P_2^*)-2(\tilde{P}_1+\tilde{P}_2) = \tilde{L}_1^{\alpha} \tilde{K}_1^{\beta}\left[z^{\alpha}-1 + (1+e^{\lambda} + 1-z)^{\alpha}(1+e^{k})^{\beta} - (1+e^{\lambda})^{\alpha}(1+e^{k})^{\beta}\right]$$

As $z < 1$, we put it $z = 1 - \epsilon$ where ϵ is a small number, then (6) is approximated to

$$= \tilde{L}_1^{\alpha} \tilde{K}_1^{\beta}\left[-\alpha\epsilon + \left(\frac{1+e^{k}}{1+e^{\lambda}}\right)^{\beta} \alpha\epsilon\right]$$

As $k > \lambda$, the quantity within brackets is definitely positive; hence the proof of the assertion that CMP approach leads to a higher level of output than IMP. It can be noted in passing that if $\lambda = k$, (6) reduces to zero, i.e., if $\tilde{L}$ and $\tilde{K}$ increase at the same rate, there is no difference in results according to the two approaches.

Turning now to the case when $\lambda > k$,

$$\frac{L_1^2}{K_1^2} = \left(\frac{\tilde{L}_1}{\tilde{K}_1}\right) z' \quad \text{where} \quad z' = \frac{1+\left(\frac{1+e^{\lambda}}{1+e^{k}}\right)^{\alpha}}{1+\left(\frac{1+e^{k}}{1+e^{\lambda}}\right)^{\beta}} > 1$$

It is obvious (from the numerical illustration if necessary) that $\tilde{K}_1$ will not be fully utilised and $\tilde{L}_1$ will be fully utilised in period 1 according to CMP approach, so that $L_1^2 = \tilde{L}_1$ and

$$K_1^2 = \frac{L_1^2 \times \tilde{K}_1}{\tilde{L}_1} \cdot \frac{1}{z'}$$

$$= \tilde{K}_1 \cdot \frac{1}{z'}$$

$$= \tilde{K}_1(1-\epsilon) \quad \text{where } \epsilon \text{ is a small number}$$

and $\tilde{\tilde{K}}_2 = \tilde{K}_2 + \tilde{K}_1 - K_1^2 \quad = \tilde{K}_1 \left\{ 1+e^k + 1-(1-\epsilon) \right\}$

as before,

$$2(P_1^* + P_2^*) = 2\left\{ \tilde{L}_1^{\alpha} \; \tilde{K}_1^{\beta}(1-\epsilon)^{\beta} + \tilde{L}_1^{\alpha} \; (1+e^{\lambda})^{\alpha} \; \tilde{K}_1^{\beta}(1+e^k + \epsilon)^{\beta} \right\}$$

and

$$2(\tilde{P}_1 + \tilde{P}_2) = 2\tilde{L}_1^{\alpha} \; \tilde{K}_1^{\beta} \left\{ 1+(1+e^{\lambda})^{\alpha}(1+e^k)^{\beta} \right\}$$

(6') $$2(P_1^*+P_2^*)-2(\tilde{P}_1+\tilde{P}_2) = 2\tilde{L}_1^{\alpha}\tilde{K}_1^{\beta}\left[(1-\epsilon)^{\beta}-1+(1+e^{\lambda})^{\alpha}(1+e^k)^{\beta}\left\{ (1+\frac{\epsilon}{1+e^k})^{\beta}-1 \right\} \right]$$

$$= 2\tilde{L}_1^{\alpha}\tilde{K}_1^{\beta}\left[-\epsilon\beta + (\frac{1+e^{\lambda}}{1+e^k})^{\alpha}\epsilon\beta \right] \text{ approximately}$$

(6') is again positive.

Thus in both the cases, i.e., when $\lambda > k$ and also when $\lambda < k$, factor allocation according to CMP approach leads to higher value of output than the IMP approach, even when we use CMP in one round only.

The above propositions can be easily generalised in the two directions that are relevant. Firstly, if the life of plant is not 2 periods but any θ periods, the situation does not change and the propositions apply with greater force, the higher the value of θ, for then

$$\frac{1+ (\frac{1+e^{\lambda}}{1+e^k})^{\alpha} + \cdots\cdots + (\frac{1+e^{\lambda}}{1+e^k})^{(\theta-1)\alpha}}{1+ (\frac{1+e^k}{1+e^{\lambda}})^{\beta} + \cdots\cdots + (\frac{1+e^k}{1+e^{\lambda}})^{(\theta-1)\beta}} = z \quad \text{when } \lambda < k$$

and $$= z' \quad \text{when } \lambda > k$$

and so z or $\frac{1}{z'} < 1$ more strongly than in the case when $\theta = 2$.

Further, as we have not put any restrictions on the initial values $\tilde{L}_1$ and $\tilde{K}_1$, the value of output when the factors are combined according to CMP approach will exceed that given by IMP approach by larger amounts, the greater the number of successive periods in which the factors are combined according to the former approach as against the latter approach.

6. Concluding Remarks

6. While demonstrating the superiority of CMP approach over the IMP approach, it should be noted that the latter ensures instantaneous full utilisation of resources, but the former does not. It may seem appropriate, therefore, to introduce some element of discounting in the former approach. It may, however, be noted that the degree of unemployment of one of the factors may be small comparable to technico-frictional unemployment that is usually observed or experienced. In the developing countries, the un- or underemployment of labour force is generally quite high and also it is not an uncommon sight to see imported machinery and equipment lying on the premises of sea-ports for two or three years before they are shifted somewhere else, which may not be their final destination for utilisation. What is meant by citing these facts, is that what is important is the maximization of output from given resources, and a slight delay in the utilisation of the one of the factors may be immaterial, for this is already occurring at a huge scale even in the absence of anything like CMP.

It has been shown above that when the fresh supplies of labour and capital grow at the same rate, the factor combination according to INP and CMP will be the same and the level of resultant output will also be the same. And if the rates of growth of labour and capital have been equal for a sufficiently long time in the past, then the neo-classical procedure, of factor combination will also not be different from the other two. But if the fresh supplies of labour and capital grow at different rates, the factor combination according to IMP will reflect the relative scarcity of the fresh supplies of factors in that period only, but will fail to reflect the changes in the relative scarcity of factors as they get accumulated. Hence the need for an alternative approach which takes into account the changing scarcity of factors.

The idea expressed here is clear intuitively as well. For instance, if a plant is going to last 25 years, it will be evidently inadvisable and inefficient to adopt a technique which only reflects the current scarcity of the fresh supplies of factors. On the contrary, the techniques should be such that they reflect the expected changes in the relative scarcity of factors over the coming 25 years.

Finally, it may be asked whether it is possible to discover any alternative method of factor combination which results in an even higher amount of output than that given by the CMP approach. The answer depends on whether there is any alternative method of combination other than that implied in competitive equilibrium which raises the level of output higher than that is possible under the condition of compeititve equilibrium. The idea of CMP is really an extension of the conditions of competitive equilibrium to a situation where the factors of production, particularly capital, are not treated perfectly malleable (as is generally required for these conditions to hold), but where they have a finite life-time once they have been created and given a shape and that they cannot be changed or transformed into an alternative shape or structure before the date of their expiry. The CMP approach is a device to overcome this rigidity and enables us to create capital items that are compatible not only with the currently available fresh supplies of labour and capital, but also with their amounts as they get accumulated over the life-time of the capital items created in the initial period.

References:

1. Jan Tinbergen , "Zur Theorie der Langfristigen Wirtschaftsentwicklung", Weltwirtschaftlishes Archiv, 1942.
2. Trygve Haavelmo, "A Study in the Theory of Economic Evolution", North-Holland Publishing Company, 1954.
3. R.M. Solow , "A Contribution to the Theory of Economic Growth", The Quarterly Journal of Economics, Vol.XLL, Feb. 1956.
4. S. Valavanis-Vail , "An Econometric Model of Growth: USA 1896-1953", The American Economic Review, Vol.XLV, May 1955.
5. Leif Johansen , "Substitution versus Fixed Production Coefficients in the Theory of Economic Growth: A Synthesis", Econometrica, Vol. 27, April 1959.

(A.H.)

6. B.F. Massel , "Investment, Innovation and Growth", Econometrica, Vol. 30, April 1962.