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Planning Strategies For Development Of New Items.

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Introduction:

In the following we deal with parallel

- Planning strategies for the development of a new item. Let us assume that several proposals for developping the new items are avaialable.

Usually one is interested to choose the best of these proposals with respect to some criterion. But it may happen that the data available. at this point of the development process are very inaccurate and uncertain and therefore unreliable.

For example a proposal promising low costs and low time to bring the project to the end may turn out to be very expensive after its completion. In such sitnations a wrong decision may be avioded by pursuing a parallel path approach. Several, say m, of the proposals are pursued to a review-point at which point the best project is selected and brought to completion while the other still remaining approaches are stopped.

This is the simplest model of parallel path-approach; generalization to several review points, however, are possible and done in the literature. (See Marschak (3)).

It is the purpose of this paper to answer the following question: How much must the uncertainty at the review point be reduced in order that a parallel-approach is at all warth while to be pursued? A lower bound on the variation of the cost-estimate is obtained in order that a parallel-approach with m proposals to start at the beginning of the project may be worthwhile at all, it is shown that this lower bound is attaned only if the cost estimates are discrets, allowing only two values.

1- A General Model

Let us have a set $\{1,2,...\}$ of Research and development approaches which can start at a point t, a set $F = \{i_1, i_2, ... i_m\}$ of m approaches of them are pursued to a review - point $t + \theta$, at which point the best project, i.e. that project having the smallest total money and time cost estimate is selected and brought to the end while the other remaining projects are stopped.

In order that such a procedure can work the cost-estimate must in some sense be consistent with the actual costs, i.e., there must be some relationship between cost-estimators and actual costs. This relationship is unbaisedness and is formulated as follows.

Let $K_{i,t+\theta}$ be the total time and money-cost that finally is obtained by bringing proposal i from review-point $t + \theta$ to completion and let moreover $K^{(i)}_{t+\theta}$ be the total time and money-estimate obtained at the point $t + \theta$.

We again assume that $K^{(i)}_{t+\theta}$ is an unbaised estimator of $K^{(i)}_{t+\theta}$ is an unbaised estimator of $K_{i,t+\theta}$ (2) , (3) i.e., that

(1) $E(K_{i,t+\theta} / K^{(i)}_{t+\theta}) = (K^{(i)}_{t+\theta})$

This assumption may seem strange since often in practice cost-estimates are much lower than actual cost. To make assumption (1) more realistic it may be assumed that

(1̃) $E(K_{i,t+\theta} / K^{(i)}_{t+\theta}) = g\,(K^{(i)}_{t+\theta})$

where g is a monotonic non-decreasing function, e.g. $g(x) = 3x$
It dose not matter in our analysis if we replace assumption (1) by the assumption (1) by the assumption (1̃) i.e. $K^{(i)}_{t+\theta}$ is replaced by $g(K^{i}_{t+\theta})$ when ever it occurs.

Let Γ be a set of approaches and let W_j be the money cost which is necessary to bring approaches $\gamma \in \Gamma$ to review point $t + \theta$. IF C_Γ is the total time and money.
- Cost which is necessary to follow a parallel-approach with approach-set Γ, then under assumption (1),

(2) $E(C_\Gamma) = \sum_{\gamma \in \Gamma} E(W_\gamma) = E\,(\min_{\gamma \in \Gamma} K^{(\gamma)}_{t-\theta})$

Indeed, let

$\gamma_0 \in \Gamma$ be such that

$$K^{(\gamma_0)}_{t+\theta} = \min_{\gamma \in \Gamma} K^{(\gamma)}_{t+\theta}$$

then by (1) :

(3) $C_{\Gamma} = \sum_{\gamma \in \Gamma} W_{\gamma} + K_{\gamma_0, t+\theta}$

(4) $E(C_{\Gamma}) = \sum_{\gamma \in \Gamma} E(w_{\gamma}) + E(E(K_{\gamma_0 : t+\theta} \setminus K^{(\gamma_0)}_{t+\theta}))$

$= \sum_{\gamma \in \Gamma} E(W_{\gamma}) + E(K^{(\gamma_0)}_{t+\theta})$

$= \sum_{\gamma \in \Gamma} E(w_{\gamma}) + E(\min_{\gamma \in \Gamma} K^{(\gamma)}_{t+\theta})$

If $\Gamma = (i_1, \ldots, I_m)$ and

(5) $F_{\Gamma}(x) = F_{i_1 \ldots i_m}(x) = P(\bigcap_{\gamma \in \Gamma} \{K^{(\gamma)}_{t+\theta} > x\})$

then by using (4) (see Marochak (3)) we get

(6) $E(c_{\Gamma}) = \sum_{\gamma \in \Gamma} E(w_{\gamma}) + \int_0^{\infty} F_{\Gamma}(x)\, dx$

suppose that the projects are ordered according to increasing expected looking costs , i.e.

$$E(w_1) \leq E(w_2) \leq \ldots\ldots$$

We now make the following assumption , The distributions of the $K^{(i)}_{t+\theta}$ are consistent with the value of $E(w_i)$. More precisely :

if $E(W_i) > E(W_j)$, we require that there is a higher probability that $K^{(i)}_{t+\theta}$ exceeds a given value x than that $K^{(j)}_{t+\theta}$ exceeds the value of x and this should hold hor any x, given an arbitrary set of alternative approaches.

Definition (4) The given set of approaches is said to have a mono - tonous closs of distribution function if for any subset of approaches Γ and pair (i, j) , $i, j \notin \Gamma$ and any real number x

(7) $P(k^{(i)}_{t+\theta} > x \mid \bigcap_{\gamma \in \Gamma} k^{(\gamma)}_{t+\theta} > x) \geq P(k^{(j)}_{t+\theta} > x \mid \bigcap_{\gamma \in \Gamma} K^{(\gamma)}_{t+\theta} > X)$

, if i j

Now under the assumptions (1) and (7) we have

a) $E(C_{\Gamma vi}) \geqslant E(C_{\Gamma vj})$

for any subset of approaches Γ if $i \geqslant j$

b) $\Gamma = \{1\}$ is the optimal set of approaches to be pursued if

(8) $\int_0^\infty P(K^2_{t+\theta} \leqslant x, K^1_{t+\theta} > x)\,dx < E(W_2)$

(c) If there is a lost integer $m \geqslant 2$ such that

$$E(W_m) \leqslant \int_0^\infty P(K^{(m)}_{t+\theta} \leqslant x, \bigcap_{i=1}^{m-1} K^{(i)}_{t+\theta} > x)\,dx$$

then $\Gamma = (1, 2, \ldots, m)$ is the optimal set of approaches to be persued. From a we see that : under the monotonity assumption it is always warthwhile to substitute a given approach by an approach possessing a lower cost estimate ; by such a substitution in the total expected time and money cost is not increases.

2 - Upper and lower bounds for the optimal number of approaches to be persued .

In this section we simplify the general model of the previous section by assuming that the cost estimates $K^{(r)}_{t+\theta}$ are stochastically independent random variables . Under this assumption the optimality- criterion simplifies to

(9) $\int_0^\infty P(K^{(m)}_{t+\theta} \leqslant x) \prod_{i=1}^{m-1} (1 - p(K^{(i)}_{t+\theta} \leqslant x))\,dx \geqslant E(W_m)$

or we define

(10) $F_i(x) = P(K^{(i)}_{t+\theta} \leqslant x)$

(11) $\int_{o}^{\infty} F_m(x) \prod_{i=1}^{m-1} (1-F_i(x))dx \geqslant E(W_m)$

The monotonity - assumption implies that

(12) $1 - F_i(x) \leqslant 1 - F_m(x)$; $i = 1, 2, \ldots\ldots m-1$

We now make the rather realistic assumption that all considered random variables (cost- estimates $K^{(i)}_{t+\theta}$ are restricted to a finite interval (M_1, M_2) i. e that

(13) $P(M_1 \leqslant K^{(i)}_{t+\theta} \leqslant M_2) = 1$, $i = 1, 2, \ldots$

This implies that

$F_m(x) = o$ if $x \leqslant M_1$

and $F_m(x) = 1$ if $x \geqslant M_2$

using this and (12) we get from (11)

(14)
$$E(w_m) \leqslant \int_{o}^{\infty} F_m(x) \prod_{i=1}^{m-1} (1 - F_i(x)\ dx$$
$$\leqslant \int_{M_1}^{M_2} F_m(x) \ . \ (1-F_m(x)\ \ dx$$

it is necessary heir to given the following well known facts.

1 - The function

(15) $f(p) = p^m(1-p)$ $o \leqslant p \leqslant 1$

is monotonously increasing for $P \varepsilon (o, m(m+1)^{-1})$ and monotonously decreasing for $p \in (m(m+1)^{-1}, 1)$ i.e

$$\max_{0 \leq p \leq 1} f(p) = f\left(\frac{m}{m+1}\right) \quad A_m = \left(1 + \frac{1}{m}\right)^{-m} \cdot \frac{1}{1+m}$$

(16) $$= \left(1 + \frac{1}{m}\right)^{-(m+1)} \cdot \frac{1}{m}$$

(17) $$((m+1)e)^{-1} \leq A_m \leq (me)$$

$$, \lim_{m \to \infty} m \cdot Am = e^{-1}$$

where $e = \lim_{m\to\infty}\left(1 + \frac{1}{m}\right)^m = \sum_{m=0}^{\infty} \frac{1}{m!} = 2{,}71818$

that means , if the $K^{\gamma}_{t+\theta}$ $\gamma = x , 2 , \ldots.$

are stochastically independant random variables and $p(M_1 \leq K^{(\gamma)}_{t+\theta} \leq (M_2) = 1$

then a necessary condition that there exist distribution function

$F_1(x), \ldots , F_2(x)$ such that

(18) $$F_i(x) = p(K^{(i)}_{t+\theta} \leq x) \leq F_{i-1}(x) = p\,(K^{(i-1)}_{t+\theta} \leq x)$$

for $i = 2 , 3, \ldots m$ and $\Gamma = i, 2, \ldots , m$

is the optimal approaches to be pursued is that

(19) $$M_2 - M_1 \geq E\,(W_m)\,(A_{m-1})^{-1}$$

(20) $$\lim_{m \to \infty} (m+1)\left(\left(1 + \frac{1}{m}\right)^m - e\right) = -\frac{e}{2}$$

which yield to the approximation

(20) $$(M_2 - M_1)\,(E\,(w_m 1)^{-1} \geq a_{m-1} \cong \left(m - \frac{1}{2}\right) e$$

where

$$a_m = (A_{m-1})^{-1} = m\,(1 + (m-1)^{-1})^m$$

and $a_m \in (m-1)e , me)$

To study the problem of (19) in the case of

$$M_2 - M_1 = (A_{m-1})^{-1}\,E\,(w_m)$$

it is necessary to make the more stringent assumption that all $K^{(i)}_{t+1}$ are not only independent but also have all the some distribution $F(x)$ but we consider K + 1 - class - parallel - planning models i.e , we assume that $F(x)$, belonge to a rondom variable X such that

$$N_1 < N_2 < \ldots\ldots < N_{k+1} \text{ and } p\,(x = N_i)$$
$$= P'_i \text{ , } i = 1, \ldots , k+1 \text{ where of course}$$
$$P_i \geqslant 0 \text{ , } 1 \leqslant i \leqslant k+1 \text{ and}$$
$$\sum_{i=1}^{k+1} P_i = 1$$

let

(21) $$P_i = 1 - \sum_{j=1}^{i} P_j \qquad i = 1, 2, \ldots\ldots , K$$

Then $F(x)$ is equal to $1 - P_i$ in the inteval $(N_i , N_{i+1}($Evidently $N_1 = M_1$ and $M_2 = N_{k+1}$

So we get from

(22) $$\sum_{i=1}^{k} M^*_i \,(1 - P_i)\, P^m_i \geqslant E\,(W_m)$$

where $M^*_i = N_{i+} - N_i$, $i = 1, 2, \ldots , k$

let us assume that $M^*_1, M^*_2, \ldots\ldots M^*_{k-1}$

as well as $P_1, \ldots\ldots , P_k$ are given what is the lowest value of M^*_k such that we can find a real number $P_k > 0$ (or equvalently a P_r such that $p_k \leqslant P_{k-1}$) with

$$\int_{M_1}^{M_2} F\,(x)\,(1 - F(x))^{m-1} dx \quad E\,(W_m)$$

...to hold ? Evidently

(23) $$H_m(P_{k-1}) = \max_{p_k \leq P_{k-1}} (1 - p_k)\, p_k^{m-1}$$

$$= \begin{cases} P_{m-1} & \text{if } P_{k-1} \geq (m-1)^{-1} m \\ (1 - P_{k-1})\, p_{k-1}^{m-1} & \text{if } P_{k-1} \leq (m-1)^{-1} m \end{cases}$$

So the stated problem has a solution if and only if

(24) $$\sum_{i=1}^{k-1} M_i^* (1 - p_i)\, p_i^{m-1} + M_k^*\, h_m(p_{k-1}) \geq E(W_m)$$

or

(25) $$M_k^*\, h_m(p_{k-1}) \geq E(W_m) - \sum_{i=1}^{k-1} M_i^* (1-p_i)\, p_i^{m-1}$$

(26) i.e. $$M_k^* \left(E(W_m) - \sum_{i=1}^{k-1} M_i^* (1-p_i)\, p_i^{m-1}\right) \left(h_m(p_{k-1})\right)^{-1} = M_{k,\min}^*$$

(27) $$M_2 - M_1 = \sum_{i=1}^{k} M_i^* \geq (h_m(P_{k-1}))^{-1} \left(E(W_m) + \sum_{i=1}^{k-1} M_i^* \left(h_m(P_{k-1}) - (1 - P_i)(p_i)^{m-1}\right)\right)$$

Specialy if $M_1^* = M_2^* \ldots\ldots\ldots\ldots = M_{k,\min}^*$ we get

$$M^* = E(W_m) \left(h_m(p_{k-1}) + \sum_{i=1}^{k-1} (1-p_i)\, p_i^{m-1}\right)^{-1}$$

and

(28) $$M_2 - M_1 \geq k M_{k,\min}^* = K\, E(W_m) \left(h_m(p_{k-1}) + \sum_{i=1}^{k-1} (1-p_i) p_i^{m-1}\right)^{-1}$$

if K = 2 we get

$$M_2 - M_1 \geq 2 \overset{*}{M}_{2,\min} = \begin{cases} 2E(W_m)\left(A_{m-1} + P_1(1-P_1)^{m-1}\right)^{-1} \\ \text{if } P_1 \leq m^{-1} \\ E(W_m)\left(P_1(1-P_1)^{m-1}\right)^{-1} \\ \text{if } P_1 \geq m^{-1} \end{cases}$$

(29) will be plotted as a function of P_1 for $m = 1$ and $E(W_m) = 1$ later on .

it may also happen that a certain strategy m_o will never apply for given values of $\overset{*}{M}_1, \overset{*}{M}_2, \ldots, \overset{*}{M}_{k-1}, P_1, P_2, \ldots, P_{k-1}$ because the strategy $m = m_o + 1$ is alawys superior to the strategy $m = m_o$, what ever may be the value of P_k. This can happen if an only if

(30) $$\overset{*}{M}_k (1 - P_k) P_k^{m_o} \geq E(W_{m_o+1}) - \sum_{i=1}^{k-1} \overset{*}{M}_i (1 - P_i) P_i^{m_o}$$

For all $P_k \leq P_{k-1}$

Specially $P_k = o$ implies

(31) $$E(W_{m_o+1}) - \sum_{i=1}^{k-1} \overset{*}{M}_i (1 - P_i)\, p_i^{m_o} \leq 0$$

if $\overset{*}{M}_1 = \overset{*}{M}_2 = \ldots\ldots = \overset{*}{M}_{k-1}$ (equidistance of cost estimates) then (29) becomes

(32) $$\overset{*}{M} \geq \left(\sum_{i=1}^{k-1} (1-p_i)\, P_i^{m_o} \right)^{-1} E(W_{m_o+1})$$

and thus

(33) $M_2 - M_1 = (k-1)\, \overset{*}{M}_{k-1} + \overset{*}{M} \geqslant (k-1)\, E\,(W_{m_o+1})$

$$\left(\sum_{i=1}^{k-1} (1-P_i)\,(p_i^{m_o})^{-1} + \overset{*}{M}_k \right.$$

if more over $\overset{*}{M} = \overset{*}{M}_k$, too and $K = 2$, we get

(34) $(M_2 - M_1)\,(\,E\,(\,W_{m_o+1})) \geqslant 2 . \overset{*}{M} \geqslant 2(P_1\,(1-P_1^{m_o})\,)^{-1}$

it will be also plotted as a function of P_1 for $m_o = 1$.

For large m(and if $P_i \geqslant (m+1)^{-1}$ in general it is twic the minimum - value of $M_2 - M_1$ for which the stragtegy m_o+1 will apply at all; while (34) gives the minimum values of $M_2 - M_1$ for which no strategy $m \leqslant m_o$ will apply. Our computions and tables will also reveal to this situation.

3 - 1 The computation of probability intervals corresponding to given optimal strategies .

Given a certain set of values $\overset{*}{M}_1, \overset{*}{M}_2, \ldots\ldots \overset{*}{M}_k$, , P_1 , P_2, ... P_{k-1} the question may arise how the probability in tervals for P_k can computed for which a given strategy $m \geqslant 2$ is optimal. The interval in which m = 1 is the optimal strategy is then evidently the interval $(o\ ,\ P_{k-1}) - \bigcup_{m=2}^{\infty} I_m$; if I_m denote the interval in which strategy m is optimal). We have evidently

(35) $P_k = P_{k-1} - P_k$

implying $P_k \leq P_{k-1}$

By (24) for the given values, P_k and hence P_k optimal if m is the last integer such that

$$(36)\quad (1-P_k)\, P_k^{m-1} \geq M_k^{*-1} \left(E(W_m) - \sum_{i=1}^{k-1} M_i^{*} (1-P_i)\, P_i^{m-1} \right) = N_m^{*}$$

If N_m^{*} happens to be smaller or equal to zero, then any $P_k \leq P_{k-1}$ satisfies the inequality (36). let us denote by J_m the set of values $P_k \leq P_{k-1}$ such that inequality (36) is met.

If $J_m = (a, b)$, then evidently $J_m^{(o)} = (P_{k-1} - b, P_{k-1} - a)$ is the P_k- interval satisfying - (36). Then

$$J_m^{(o)} \supseteq J_{m+1}^{(o)} \quad \text{and}$$

$$(37)\quad I_m = J_m^{(o)} - J_{m+1}^{(o)}$$

We know from the last paragraph that there is a lower bound for N_m^{*} in order that $J_m^{(o)} \neq \phi$, namely that

$$(38)\quad N_m^{*} \leq h_m(P_{k-1}) = \begin{cases} A_{m-1}, & \text{if } P_{k-1} \quad (m-1)\, m^{-1} \\ P_{k-1}^{m-1}(1-p_k) & \text{if } P_{k-1} \leq (m-1)^{-1} m \end{cases}$$

If $N_m^{*} \leq o$ then evidently $J_m = (o, P_{k-1})$,

$$J_m^{(o)} = (o, P_{k-1}) = (o, 1 - \sum_{i=1}^{k-1} P_i)$$

now let us assume that $o < N_m^{*} < h_m(P_{k-1})$

if $N^*_m = h_m (P_{k-1})$, then $P_k = P_{k-1}$ is the only element of $J^{(o)}_m$ in which case the k+1 class- parallel planning - model degenerates to a k - class- parallel- planning model) We now make use of the results of the lost poraggraph, nomely that $g_{m-1}(p) = (1-p)p^{m-1}$ is monoto wsly increasing in $(o , m^{-1}(m-1)$ and monotonously increasing in $((m-1) . m^{-1})$

The maximum is at $P = (m-1)\ m^{-1}$ a saddle - point at $(m-2)\ m^{-1}$

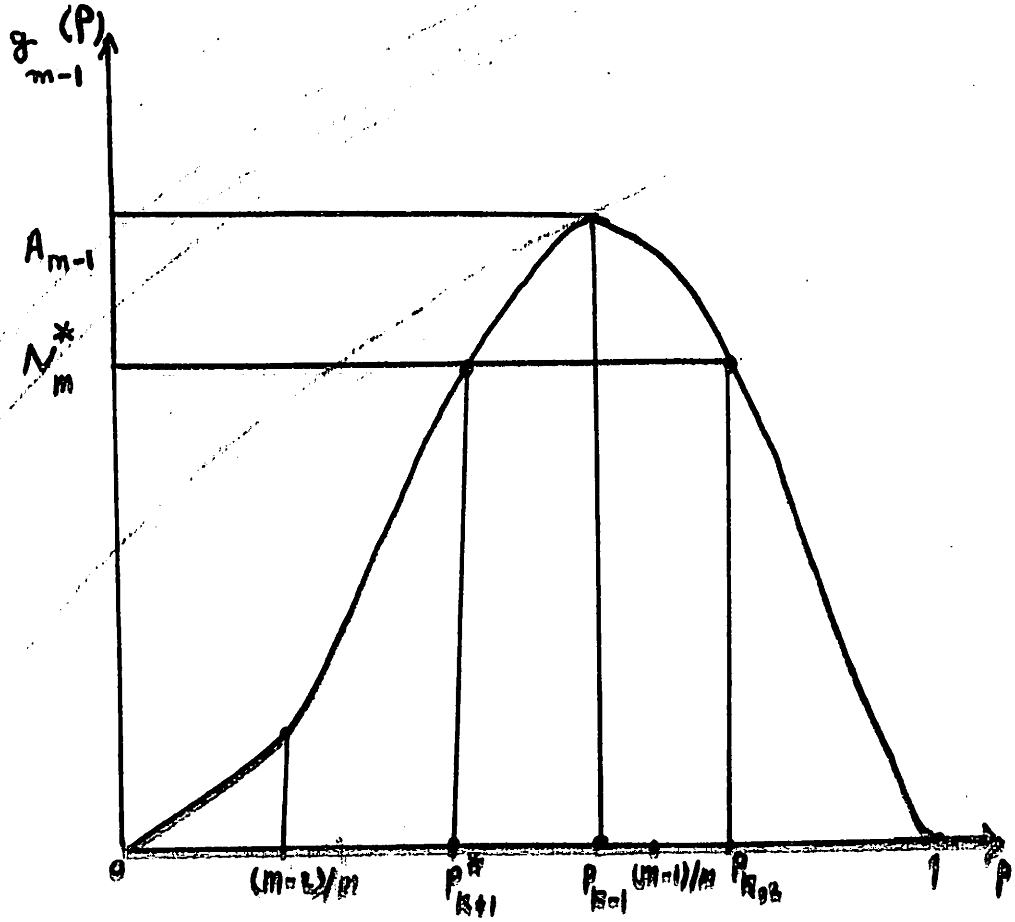


Fig 1 = Situation A

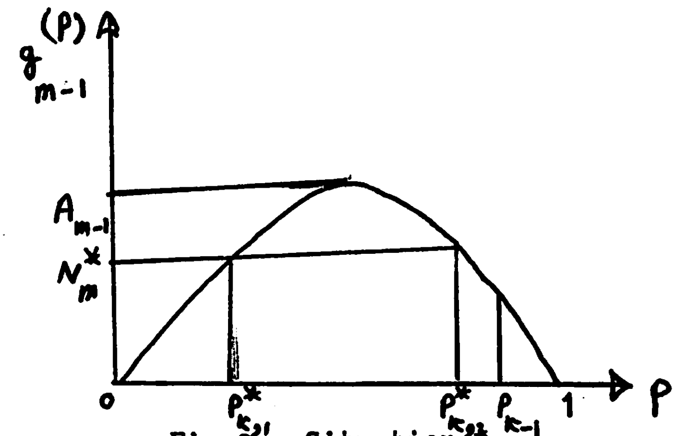


Fig 2 - Situation B

If $o < N_m^* < h_m(P_{k-1})$, there will exist - digram and analysis of $g_{m-1}(p)$ show it - two values $P_{k,1}^*$, $P_{k,2}^*$ such that $P_{k,1}^* < P_{k,2}^*$ and there fore $(1-P_{k,2}^*)\,(P_{k,2}^*)^{m-1} = (1-P_{k,1}^*)\,(P_{k,1}^*)^{m-1} = N_m^*$ thus

Thus

$$(1-P_k)\,P_k^{m-1} > N_m^* \text{ if } P_k \in (P_{k,1}^*, P_{k,2}^*) .$$

$$N_m^* < h_m(P_{k-1} \text{ implies } P_{k,1}^* < P_{k-1}$$

In situation A (fig 1). $P_{k,2}^* > P_{k-1}$ hence

(39) $J_m = (P_{k,1}^* , P_{k-1})$, $J_m^{(o)} = (o, P_{k-1} - P_{k,1}^*)$

In situation B (lig 2)

$$P_{k,2}^* > P_{k-1} \text{ and hence}$$

(40) $J_m = (P_{k,1}^* , P_{k,2})$, $J_m^{(o)} \triangleq (P_{k-1} - P_{k,2}^*, P_{k-1} - P_{k,1}^*)$

In general we have

(41) $J_m = (P_{k,1}^*, \min(P_{k-1}, P_{k-2}^*))$

(42) $J_m^{(o)} \triangleq (P_{k-1} - \min(P_{k,2}^* , P_{k-1}) , P_{k-1} - P_{k,1}^*)$

$$= (\max(o, P_{k-1} - P_{k,2}^*) , P_{k-1} - P_{k-1}^*)$$

3 - 2 Computing procedure

Compute N^*_m starting with m = 2 and $J^{(o)}_m$ according to

$$
(43) \quad J^{(o)}_m \begin{cases} \phi & \text{if } N^*_m > h_m(P_{k-1}) \\ (o) & \text{if } N^*_m = h_m\,(P_{k-1}) \\ \max\,(o,\, P_{k-1} - P^*_{k,2})\,,\; P_{k-1} - P^*_{k-1}) & \\ & \text{if } o \quad N_m \quad h_m\,(P_{k-1}) \\ (o,\, P_{k-1}) & \end{cases}
$$

where $P^*_{k,1}$ denotes the smaller and $P^*_{k,2}$ denotes the larger of the two roots of the equation $P^{m-1}(1-P) = N^*_m$, provided

$o < N^*_m < 1$. Jf $J^{(o)}_m \neq \phi$

go to m+1 , jf $J^{(o)}_m = \phi$ stop the compution .

If $m = m_o$, then

$$
(44) \quad I_{m_o} = \phi \;,\; I_{m_o-1} = J^{(o)}_{m_o-1} \,;
$$

$$
I_m = J^{(o)}_m - J^{(o)}_{m-1} \qquad m = 2,\, 3\,,\, \ldots\ldots,\, m_o - 2
$$

$$
I_1 = (o,\, P_{k-1}) - \bigcup_{r=2}^{m_o-1} I_r
$$

The only thing that now must be still done is the compution of the two roots of the equation $P^{m-1}(1-p) = P^*_m$ provided

$$
o < N^*_m < A_{m-1}
$$

This problem will be solved by considening the following thearm (4)

a) let

$$A_{m-1} = g_{m-1}\left(\left(m-2\right)m^{-1}\right) = 2\,m^{-1}(1-2\,m^{-1})^{m}$$

and

$$p_o = (m-2)\,m^{-1} \text{ if } o < N_m^* \leq \hat{A}_{m-1},$$

$$P_o = (m-1)\,m^{-1} \text{ if } A_{m-1} > N_m^* > \hat{A}_{m-1}$$

and more over

(45) $P_{n+1} = (m-1)^{-1}\left[(m-2)\,P_n + N_m^* \left\{P_n^{m-2}\,(1-P_n)^{-1}\right\}\right]$

$,n = o, 1, 2, \ldots$ Then

$$P_n > P_{n+1} \text{ and } \lim_{n\to\infty} P_n = P_1^*$$

where P_1^* is the smaller of the two roots of the equation

$$P^{m-1}(1-P) = N_m^*$$

(b) let $\hat{P}_o = (m-1)\,m^{-1}$ and

(46) $\hat{P}_{n+1} = 1 - N_m^*\,\hat{P}_n^{-(m-2)}$, $n = o, 1, 2, \ldots\ldots$

then

$$\hat{P}_{n+1} > \hat{P}_n \text{ and } \lim_{n\to\infty} \hat{P}_n = P_2^*,$$

the larger of the two roots of the equation $p^{m-1}(1-p) = N_m^*$

de tailed proof in (1)

We have the following error bounds for a : ...

$P_n - P_1^*$ $(P_{n+1} - P_n)\;h_o\,(P_n)$

if $p_o = (m-1)\,m^{-1}$

$P_n - P_1$ $(P_{n+1} - P_n)\;h_1\,(P_n)$ if

$P_o = (m-2)\,m^{-1}$

Where

$$h_o(P_n) = (1 - P_n)\,(m-1) - m\,P_n)^{-1}$$

$$h_1(P_n) = (1 - P)^2((m-1) - mP_n)^{-1} \cdot P_{nn}^{m-1}(N^*_m)^{-1}$$

For (b)

$$P^*_2 - \hat{P}_n \qquad (\hat{P}_{n+1} - P_n)\,h_2\,(\hat{P}_m)$$

Where

$$b_2(\hat{P}_n) = \hat{P}_n(m\,\hat{P}_m - (m-1))^{-1}$$

4 - Grophical and Numerical Illustrations

In (20) it had been shown that a parallel - approach with m approaches pursued to review - point can be optimal strategy if

$$(M_2 - M_1)\,(E\,(W_{m_o}))^{-1} \geqslant a_{m_o} - 1 \simeq (m_o - \tfrac{1}{2})\,e$$

where $M_2 - M_1$ is the variation of the cost - estimate and $E(w_{m_o})$ the expected cost of carrying approach m to the review - point .

more over, we know that $a_{m-1} \in (\,(m-1)\,e\;\; me\,)$.

In (table 1) a_{m_o} and $(m_o - \frac{1}{2})\,e$ as well as $(m_o - 1)\,e$ and $m_o e$ are listed it turns out that even for $m_o = 2$ the error between a_{m_o} ($\neq 4$) and $(m_o - \frac{1}{2})\,e$ is less than 2 . It may also be noted that computing a_{m_o} by logarith was according to the formula .

$$a_{m_o} = \exp\,(\,m_o \log m_o - (\,m_o - 1\,)\log(\,m_o - 1\,)\,)$$

It has shown in section 2 that if

$a_{m_o} = (M_2 - M_1)/E(W_{m_o})$, then there is only one distribution function $F(x)$, concentrated in (M_1, M_2), such that $m = m_o$ can be the optimal number of approaches to be pursued, nomely the distribution which takes as values only $X = M_1$ (with probability m_o^{-1}) and $X = M_2$ (with probability $1 - m_o^{-1} = (m_o - 1)\, m_o^{-1}$

if this very special two class problem ean be excluded, then there must be a much higher value of $(M_2 - M_1)/E(W_{m_o})$ in order that $m = m_o$ can act as optimal strategy.

To win a further insight insight into the nature of this problem, let us consider that we have an equidistant three - class parallel - strategy model. i. e that $P(x = M_1) = P_1$, $P(x = M_1 + (M_2 - M_1)/2) = P_2$

$$P(x = M_2) = 1 - P_1 - P_2, \text{ where}$$

$P_1, P_2 \geqslant o$.

If $P_1 > o$ is given then necessary for the existence of some $P_2 \geqslant o$ such that $m = m_o$ may act as optimal strategy is that see(29)

$$(47)\quad (E(W_{m_o}))^{-1}(M_2 - M_1) \geqslant h_{m_o}(P_1) \quad \begin{cases} 2 O_{m_o}(1 + a_{m_o}(p_1(1-P_1)^{m_o-1})^{-1} & \text{if } P_1 \leq m_o^{-1} \\ (P_1(1-P_1)^{m_o-1})^{-1} & \text{if } P_1 \geqslant m_o^{-1} \end{cases}$$

Note that if $P_1 \geqslant m_o^{-1}$ and the equality - sign holds in (47), then $P_2 = 0$ is the only value meeting the required condition, so that the

a_{m_o} / m_o	$(m_o-1)e$	a_{m_o}	a_{m_o} togather camp	$(m_o - \frac{1}{2})e$	$m_o e$	relative error of $(m_o - \frac{1}{2})e$
2	2,718	4	3,999	4,077	5,436	2%
3	5,436	6,75	6,735	6,795	8,154	1%
4	8,154	9,481	9,575	9,8513	10,872	3%
5	10,872	10,872	12,095	12,231	13,950	2%
6	13,590	13,590	14,933	14,949	16,308	1,3%
7	16,308	16,308	17,64	17,667	19,026	0,9%
8	19,026	20,371	20,375	20,385	21,744	0,7%
9	21,744	23,091	23,07	23,103	24,462	0,5%
10	24,462	25,811	25,835	25,821	27,180	0,4%
11	27,180	28,531	28,539	28,535	29,898	0,3%

Table 1: Minimal number of $(M_2 - M_1)E\ (W_{m_o})$, ratioof cost-estimates Range and expected inspection cost, that is necessary in order $m = m_o$ can be the optional number of approaches to be persued .

problem degenerates in this case to a two class problem, if $P_1 \rightarrow 1$, then $h_{m_o}(P_1) \rightarrow \infty$

This is quite clear because if $P_1=1$, then we have a one-class problem in which all uncertainly is removed and some in this case there will be no need for aparallel-strategy $h_{m_o}(P_1)$ has minimum at $P_1=m_o^{-1}$ with minimum value $h_{m_o}(m_o^{-1}) = m_o^a$

But the corres ponding value of P_2, if $(E(W_m))^{-1}(M_2 - M_1) = m_o^a$ is $P_2 = 1 - m_o^{-1}$ implying

$P(x = M_2) = o$.

So the problem degenates ti a two-class problem. The some holds if $P_1=o$: value $h_m(o)-2\, a_{m_o}$ is not really correct since in this case the distribution is concentrated in $((M_1 - M_2)/2, M_2)) = (\hat{M}_1^*, \hat{M}_2^*)$ and so $(\hat{M}_2 - \hat{M}_1)/E(W_{m_o})) \geqslant a_{m_o}$ must hold in order that $m = m_o$ can act as optimal strategy.

As we can see from (fig 3) $h_{m_o}(P_1)$ has been plotted for $m_o = 2$ also the curve I_{m_o} is plotted this curve is equal to

(48) $I_{m_o}(P_1) = 2(P_1(1 - P_1)^{m_o-1})^{-1}$

and in minimal number of $(M_2 - M_1)/E(W_{m_o}))$ given P_1, in order that the strategy $m = m_o - 1$ will be never applied as optimal srtategy what ever is the value of P_2.

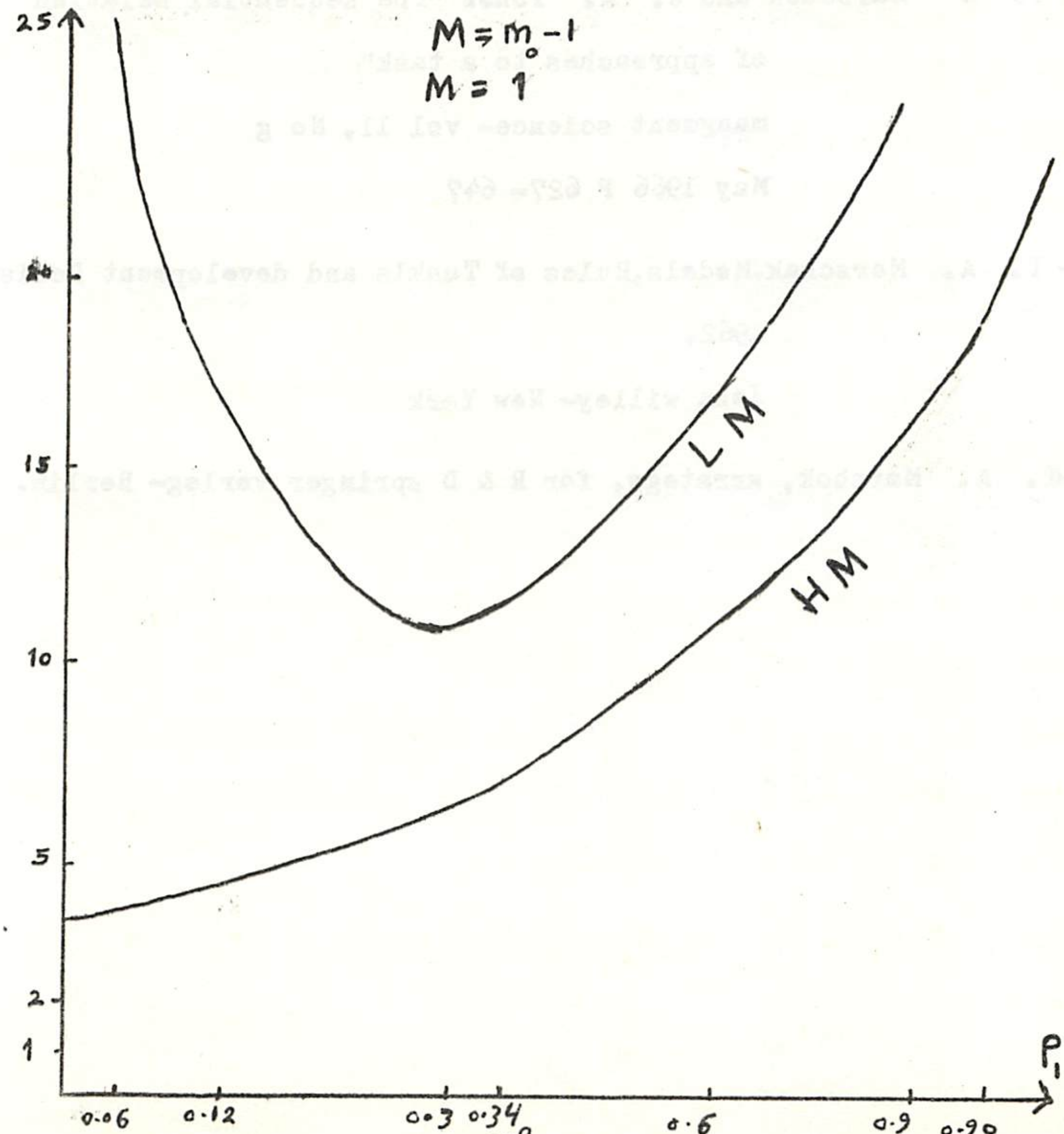
If $(M_2 - M_1)/E(W_{m_o}) \geqslant i_{m_o}(P_1)$ then either $m = m_o$ or $m = m_o + 1$, $m_o + 2$ ....

will be applied as aptimal strategy.

If $P_1 \geqslant m_o^{-1}$ then $I_{m_o}(P_1) / h_{m_o}(P_1) = 2$

If $P_1 < m_o^{-1}$ this rario will be much larger and approach infinity if P_1 approach zero. This is quite clear for if $P_1 = o$.

the problem again degenrates to a two- class problem



(Fig.3)

Move matevial can be found in (4)

Reference

1- R. R. Nelson:"The economics of parallel R and D efforts: A sequential- Descision analysis" rand research Memo- 2482

2- T. A Marschok and J. A. Yohow "The sequential selation of approaches to a task" mangment science- vol 11, No g May 1966 P 627- 647

3- T. A. Marschak.Models,Rules of Tunkls and development Decisions 1962. John willey- New York

4-T. A. Marshok, srrategy, for R & D springer verlog- Berlin.