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PART PERIOD ALGORITHM

by

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INTRODUCTION:

The objective of inventory management is to maintain optimum levels of inventory consistent with customer demands and plant capacity. Management must determine:

- What to order
- When to order
- How much to order

This is not an easy task, for there are many conflicting goals. Still management must inevitably make a decision to order what it considers to be economical.

The determination of an economic ordering quantity is commonly referred to as "Lot-Sizing". Calculations of the size of an order yield to the total cost as composed of the following elements:

- Ordering costs, including setup
- Inventory holding costs.

As a lot-sizing technique for minimizing the sum of ordering and inventory costs, the author is going to handle here a new concept, which was first developed by J.J. De Matteis* and presented here with some modifications, based on some simple dimensions. By dividing the ordering and setup costs by the inventory holding costs per pant pen time period, the ordering costs are expressed using a new dimension which we will call(part-periods). This value is used to determine lot size.

First of all, a simplified version is shown for demand sets that do not vary widely between periods. For large variations in demand, significantly greater overall accuracy is achieved with look-ahead and look-back tests which are also included. A detailed flow-chart for the accurate procedure together with a FORTRAN II coded program are presented. An illustration is also given.

To get the picture complete, two of the more important economic lot-sizing algorithms are compared with the Part Period Algorithm.

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* John J. De Matteis, Advanced systems Development Division, IBM; N.Y.

THE ALGORITHM:

The part-period concept is based on the following simple considerations:

- If one part (one unit) is held in inventory for one period, it incurs a particular holding cost.
- If one part is held for two periods, it incurs twice the holding cost.
- Two parts held for one period incur the same cost as one part held for two periods.
- Two parts held for three periods incur the same as three parts held for two periods or as one part held for six periods or as six parts held for one period, etc.
- If the number of parts held in inventory are multiplied by the number of periods over which they are held, the dimension "part-period" is derived. Expressing the ordering costs in this new dimension, a simple and effective lot-sizing technique is obtained.
  Now, let's see how can we do that. Simply, be dividing the ordering costs by the inventory holding costs per part per period, ordering costs are expressed in part periods.
  If; for example:
  The ordering cost, for any number of prices of a certain part, is $50 & The inventory holding cost is $0.50 per unit per period.
  ∴ In this case we have a part period value of 50/0.50= 100
- Once ordering costs for each part number are converted to part-periods, this simple calculation need be done again only if there is a change in material and labor costs, setup, inventory holding, etc.

THE SIMPLIFIED VERSION OF THE ALGORITHM (or PHASE I)

This version is very effective for all except the most demanding circumstances, and also outperforms the Wagner-Whitin Algorithm in the short-horizon environment (this point will be extensively discussed later on)
In order to see how does work proceed with this algorithm, assume:

- The part-period value of a particular part number to be 100
- The demand by periods to be $d_1, d_2, \ldots\ldots\ldots d_n$
- No holding costs are incurred for items consumed in the period in which they are ordered.

To determine the reorder point and reorder quantity, based on the previous assumptions, we proceed as follows:

- We compute the cumulative part-period values as follows:
  $(0)\ d_1 + (1)\ d_2 + (2)\ d_3 + (3)\ d_4 + \ldots\ldots\ldots$
  until this values exceeds the part-period value for this particular part (which is 100 in our particular case)

- The setup should be made in the period that causes cumulative value to exceed 100
- The reorder quantity is then the sum of the demands for the periods covered by the order.
- Concerning cost, the cost of inventory keeping alone is considered if there is no setup. If there is setup, obviously consider the cost of setup only Computation of costs are not required for our analysis here; but they are included here for comparison with other algorithms later on.

<u>An Illustrative Example:</u>

Make the following assumptions:

- Setup cost = $ 50
- Inventory holding cost = $ 0.5 per part per period
- The part-period value for this part = 100
- The demand for each period is as follows:

PERIOD	1	2	3	4	5	6	7	8
DEMAND	20	20	25	35	30	10	10	15

How can we proceed? (Please Refer to Table I)

1. A setup is made in Period 1
2. The demand units for Period 1 is consumed in the same period in which it is produced and will therefore incur no part-period costs.
3. The demand for Period 2, if ordered in Period 1, is held for one period incurring a holding costs of 1x20 or 20 part-periods. ∴ 20 is deducted from the 100 part-periods available for the items, leaving a balance of 80 part-periods.
4. In a similar manner, the demand for Period 3 is held two periods, incurring a cost of (2x25) or 50 part-periods This leaves a new balance of 30 part-periods.
5. For period 4, the demand is held for three periods, incurring a cost of (3x35) or 105 part-periods It is obvious that those requirements outbalance the 30 available part-periods, so we need a setup in Period 4
6. The order quantity for the first setup (at Period 1) is to be enough to cover the demand for periods 1 till the start of Period 4 (at least) leading us to an order size of 65 items.
7. The demand for period 5 = 1 x 30 = 30 part-periods (100)

8. The demand for Period 6 = 2 x 10 = 20 part-periods
9. The demand for period 7 = 3 x 10 = 30 part-periods
   the cumulative part-periods value = 80
10. If we try to satisfy the demand for Period 8, which is 4 x 15, we will find the supply inadequate
    This leads to a third setup in period
11. The order quantity for the second setup (at Period 8) is to be enough to cover the demand for Periods 4 till the start of Period 8 leading to an order size of 85 items.

This simple example, beside serving as an illustrative tool to explain the simplified version of the Part-Period Algorithm, it helps us to deduce two very important remarks:
- This is a variable-order-quantity system
- And also a variable-order-point system

which provides the flexibility required for computing economic lot sizes.

PERIOD	DEMAND	PART PERIOD VALUE	CUMULATIVE PART PERIODS	CUMULATIVE COST
1	20	0	0	50
2	20	20	20	60
3	25	50	70	85
4	35	105	175	135
5	30	30	30	150
6	10	20	50	160
7	10	30	80	175
8	15	60	140	225

TABLE I

THE ACCURATE VERSION (or PHASE II & PHASE III)

This is a refined version of the algorithm which yields significantly overall accuracy and reduces, or rather eliminates, the majority of the larger errors that creep into the simple version. This version is adaptable to the cases of large fluctuation in demand.

The seeked accuracy is generally accomplished through the addition of three steps, of calculation, per planning cycle.

What we do really is go through exactly the same procedure for the simplified version (or Phase I). However, once the decision to set up has been made, we:

1. LOOK AHEAD at a minimum of two periods following the tentative setup period to determine whether we will be faced with a large demand in the near future or not
   In this case the part-period value for the setup period is compared with the demand of each the periods next to it (before next setup), if is less than or equal to one of those demands, the setup decision is moved to the corresponding period. If it is not less than or equal to the demand, the original decision stands.
2. LOOK BACK to see it the demand for the previous period is also very large. Again saying, the demand for the setup period is compared with the demand of the previous period as follows:
   If $2\ d_s \leq d_{s-1}$

The setup should be in the previous period, otherwise the original decision stands as it is.

We have to notice that if the LOOK-AHEAD test succeeds in moving the setup decision one period ahead, the LOOK-BACK test should not be invoked.

The look-ahead and look-back tests help us to avoid the costly mistake of remaining on one level with a mountain of demand at either or both sides.

If we take the previously presented as an illustration (see TABLE 1), we should proceed as follows:

1. Follow the PHASE I procedure to get the result as shown in TABLE 1
2. Make the look-ahead test:
   - There is a decision for setup at Period 4, if we compare the part period value for Period 4 with demands of Periods 4 to 7, we find that it is not less than or equal to any of them
   ∴ No improvement is possible in this case

3. Make the look-back test:
   - For Period 4 ; $d_4 = 35 \quad \therefore 2d_4 = 70$
     $d_3 = 25$
     $\therefore 2d_4 > d_3$

   $\therefore$ No improvement is possible since the condition $2d_5 \leq d_{5-1}$ is not existing.

The final results are shown in the following table

PERIOD	DEMAND	PART PERIOD VALUE	CUMULATIVE PART PERIODS	TENTATIVE CUMUL COSTS	FINAL CUMUL COST
1	20	0	0	50	50
2	20	20	20	60	60
3	25	50	70	85	85
4	35	105	175	135	135
5	30	30	30	150	150
6	10	20	50	160	160
7	10	30	80	175	175
8	15	60	140	225	225

REMARKS:

It is obvious that for a horizon equal to one part-period planning cycle, PPA (Part Period Algorithm) is invariably optimal. For longer horizons, PPA yields suboptimal results; its precision variance, however is so low as to be of no consequence in a practical application.

ADVANTAGES OF PPA:

1. It is an open-ended algorithm; that is, it performs as well over most horizons as over long horizons.
2. It is highly accurate for demands with large variations between periods, as well as for those with very small variations; it is particularly outstanding in the typical industrial environment where the demand may be known for only a maximum of six or seven months.
3. It is inexpensive and quick to maintain and implement.

GENERAL CONSIDERATIONS FOR LOT-SIZING TECHNIQUES:

A large number of economic lot - rising techniques are available to management any one or combination of which may be incorporated in an inventory control system.

In choosing a lot-rizing technique, the cost of applying the technique and its performance for certain demand characteristics are the major considerations. It is acceptable that low-cost items are controlled by a technique different from that used for high-cost items. The factors to be considered when chooding a technique, or combination of techniques, include :

1- The cost and time of applying the algorithms
2- The demand range, i.e.the variability of demand from one period to another.
3- The availability of long-range forecost and confidence in the accuracy of the forecost.

Now we are going to discuss, briefly, two of the more important lot-rizing algorithms and then compare them with PPA

THE WAGNER-WHITIN ALGORITHM (1)

This algorithm is famous for producing exact solutions, for known demands, over a horizon equal to the life of the item. Rather than to test every combination of either ordering or not ordering in each period, this algorithm is able to achieve the same results in substantially fewer steps.

Two important conclusions resulted from repeated application of the algorithm:
1. The actual number of steps required varies with the nature of the demand (i.e. fluctuation manner).
2. The ratio of the setup and the inventory holding costs
3. The number of periods in the horizon.

For short horizons, application of this algorithm results in sōlutions inferior to those obtained through applying PPA

WAGNER-WHITIN ALGORITHM VS. PPA

Drawbacks of WWA in comparison with PPA are:
- Needs relatively large number of calculations in a typical environment
- Computational time is extremely sensitive to the frequency of setups
- Typically, from ten to thirty times as many calculations are required for the WWA as for PPA

(1) See H.M. Wagner and T.M. Whitin "Dynamic version of the economic lot size model" Management Science 5, No. 1 (October 1958)

THE LEAST UNIT COST ALGORITHM[1]

This algorithm tends to compute, for various order sizes the cost per unit chargeable to setup and to inventory holding and selects the minimum value

For known demand in long horizon enviroment, applying this algorithm results in a rise of total cost, over that obtained through the application of PPA, equals 5%. While for an environment of wide variations in demand the rise is 20%.

COMPARISON BETWEEN THE THREE ALGORITHMS:

An extensive analysis was performed by J.J. De Matties in his original work. He proved that the PPA performs well in all industries, but is particularly well suited for industries whose demand forecast extends for a limited no. of periods, and for those whose forecast is appreciably more accurate in the near future than for the more distant future.

In the short-horizon environment, PPA outperforms other algorithms previously refered to.

Considerably few computations are required by PPA in comparison with other algorithms with competing performance.

In the "known-demand, long-horizon" environment, setup and holding-cost performance with PPA is approximately half of one percent higher than minimum cost.

COMPUTER PROGRAMMING:

A detailed flow-chart for the PPA algorithm was constructed and then translated to a FORTRAN II coded program, and are both included hereafter.

The given example was used as test data. Also, two other examples are presented to explain the role of look-Ahead and Look-Back tests. The results are all shown.

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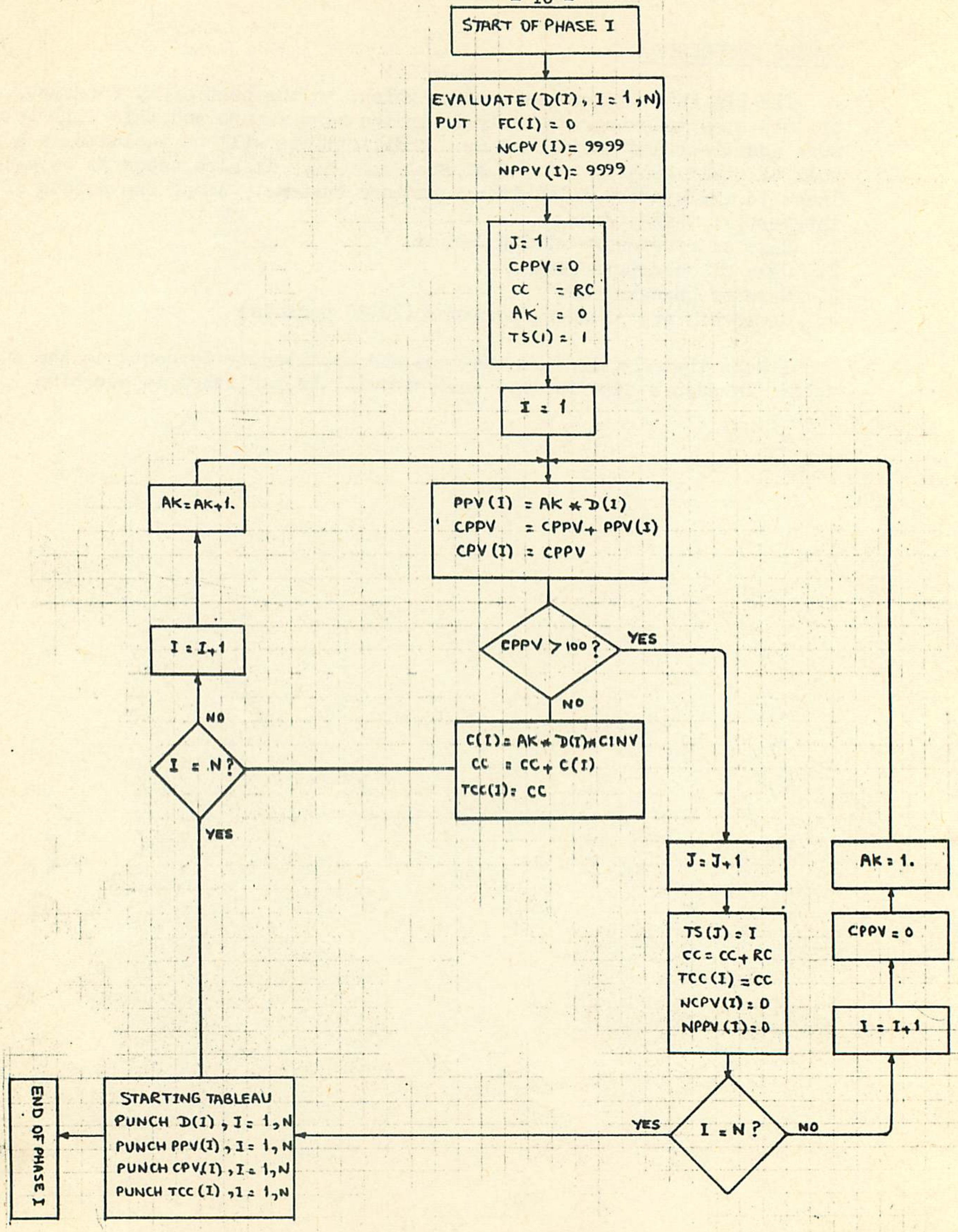
(1) For more details refer to PROINCOS, IBM reference no. E20-0280-1

FUTURE INTERESTS:

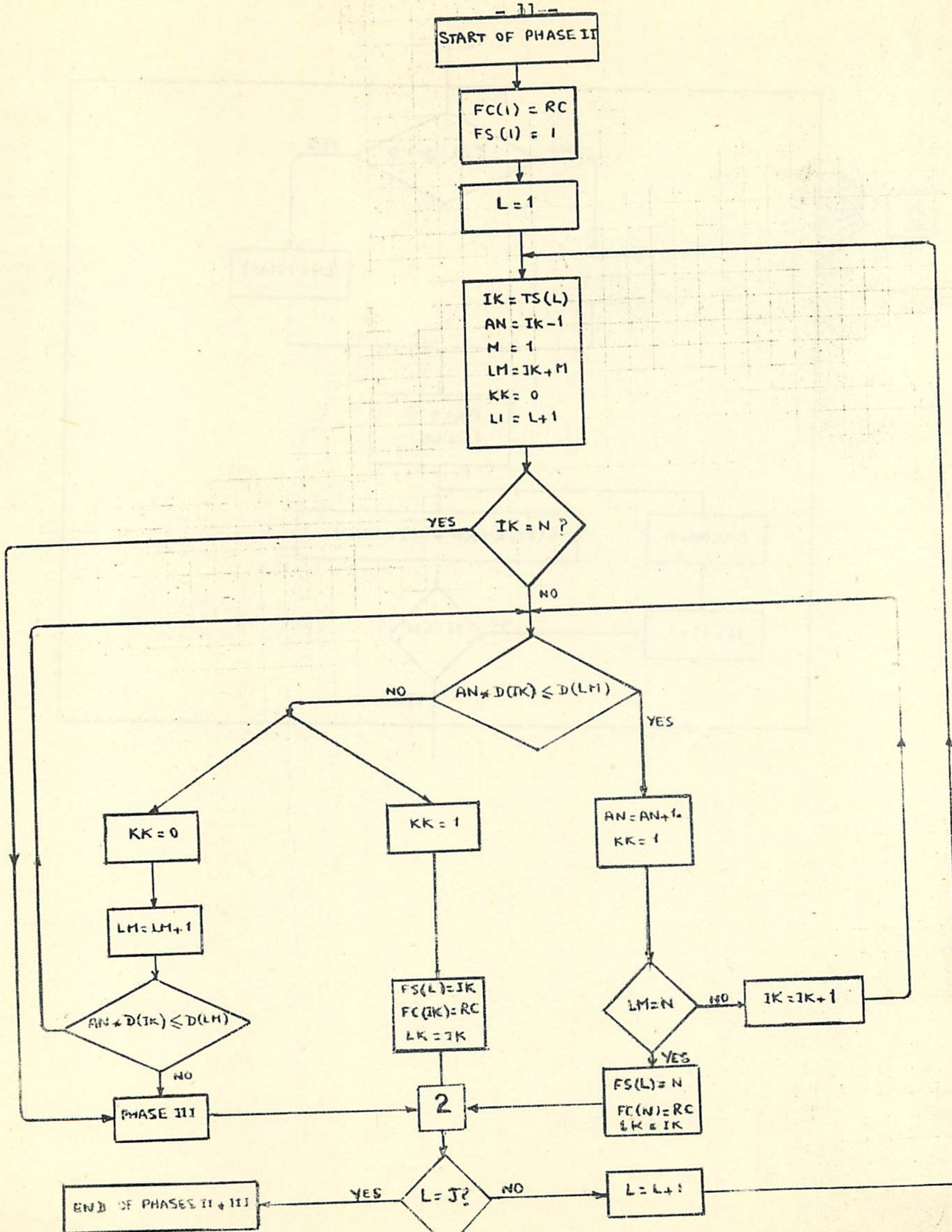
The PPA is a rather new concept which, to the best of my forecast, will be subjected to extensive analysis in the near future and this will lead to more additions and modifications. Modifications will be included as a result of field studies and actual applications. It also needs to be generalized to include the multi-item inventory systems. Among the points of interest, I think, are:

1. Case of constantly rising demand
2. Case of uncertain demand
3. Case of changing PPV
4. Cases of long planning horizons (20-30 periods)

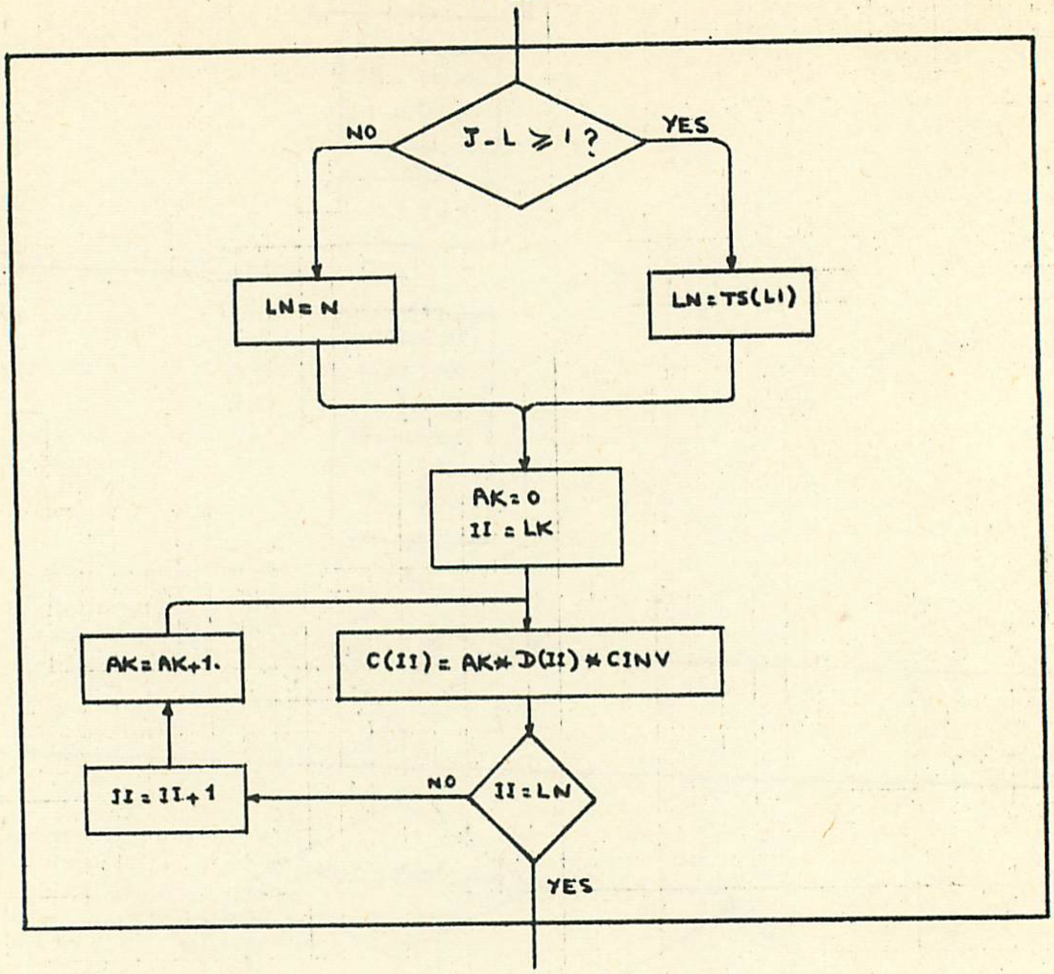
I hope that I will have the time and opportunity to continue the subject. In such a case further analysis will be published in due time.



START OF PHASE I
EVALUATE (D(I), I = 1,N)
PUT FC(I) = 0
NCPV (I) = 9999
NPPV (I) = 9999
J = 1
CPPV = 0
CC = RC
AK = 0
TS(1) = 1
I = 1
PPV (I) = AK * D(I)
CPPV = CPPV + PPV(I)
CPV (I) = CPPV
CPPV > 100 ?
YES
NO
C(I) = AK * D(I) * CINV
CC = CC + C(I)
TCC(I) = CC
I = N ?
NO
YES
I = I + 1
AK = AK + 1.
J = J + 1
TS (J) = I
CC = CC + RC
TCC (I) = CC
NCPV(I) = 0
NPPV (I) = 0
I = N ?
YES
NO
I = I + 1
CPPV = 0
AK = 1.
STARTING TABLEAU
PUNCH D(I), I = 1,N
PUNCH PPV(I), I = 1,N
PUNCH CPV(I), I = 1,N
PUNCH TCC (I), I = 1,N
END OF PHASE I

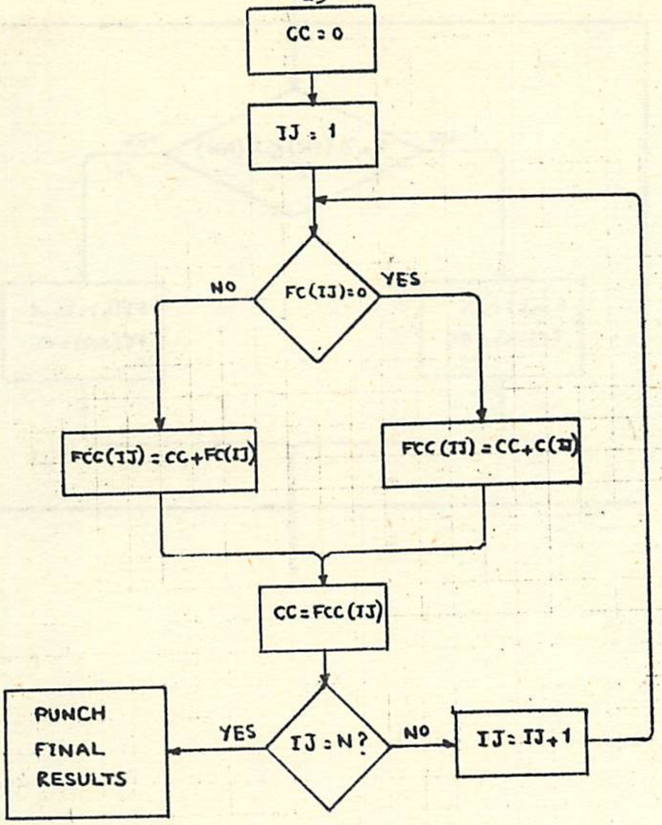

START OF PHASE II
FC(1) = RC
FS(1) = 1
L = 1
IK = TS(L)
AN = IK - 1
M = 1
LM = IK + M
KK = 0
L1 = L + 1
IK = N ?
YES
NO
AN * D(IK) ≤ D(LM)
NO
YES
KK = 0
KK = 1
AN = AN + 1
KK = 1
LM = LM + 1
FS(L) = IK
FC(IK) = RC
LK = IK
LM = N
NO
IK = IK + 1
AN * D(IK) ≤ D(LM)
YES
NO
FS(L) = N
FC(N) = RC
LK = IK
PHASE III
2
END OF PHASES II + III
YES
L = J?
NO
L = L + 1

2

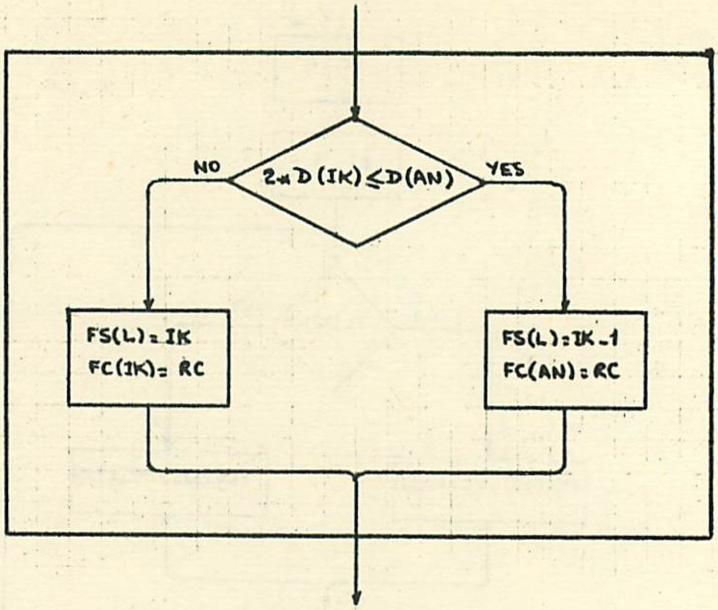


J-L ≥ 1 ?
NO
YES
LN = N
LN = TS(LI)
AK = 0
II = LK
AK = AK+1.
C(II) = AK*D(II)*CINV
II = II+1
NO
II = LN
YES

A


CC = 0
IJ = 1
FC(IJ)=0
NO
YES
FCC(IJ) = CC+FC(IJ)
FCC(IJ) = CC+C(II)
CC=FCC(IJ)
PUNCH
FINAL
RESULTS
YES
IJ = N?
NO
IJ = IJ+1

PHASE III



```
      DIMENSIONC(30),TS(30),PPV(30),CPV(30),C(30),TCC(30),NCPV(30)
      DIMENSIONNPPV(30),FC(30),FS(30),FCC(30)
  500 READ1,N
    5 READ2,(D(I),I=1,N)
      READ2,RC,CINV
    2 FORMAT(10X,10(F7.2)/10X,10(F7.2)/10X,10(F7.2))
    1 FORMAT(I3)
C
C     START OF PHASE I
C
      DO10I=1,N
      FC(I)=0
      NCPV(I)=9999
   10 NPPV(I)=9999
C
      J=1
      CPPV=0
      CC=RC
      AK=0
      TS(1)=1
C
      DO40I=1,N
      PPV(I)=AK*D(I)
      CPPV=CPPV+PPV(I)
      CPV(I)=CPPV
      C(I)=AK*D(I)*CINV
      IF(CPPV-100.)20,20,30
   20 CC=CC+C(I)
      TCC(I)=CC
      IF(I-N)25,50,50
   25 AK=AK+1.
      GOTO40
   30 J=J+1
      TS(J)=I
      CC=CC+RC
      TCC(I)=CC
      NCPV(I)=0
      NPPV(I)=0
      IF(I-N)35,50,50
   35 CPPV=0
      AK=1
C
   40 CONTINUE
C
C     PUNCH STARTING TABLEAU
   50 PUNCH3
      PUNCH4
      PUNCH6
      PUNCH7
      PUNCH8
      DO 55 I=1,N
   55 PUNCH9,I,D(I),PPV(I),CPV(I),TCC(I)
      PUNCH3
      PUNCH11
C
C     END OF PHASEI
C
C     START OF PHASE TWO
C
      FC(1)=RC
      FS(1)=1
C
      DO150L=2,J
      L1=L+1
      IK=TS(L)
      AN=IK-1
```

```
      M=1
      LM=IK+M
      KK=0
      IF(IK-N)60,100,60
   60 AND=AN*D(IK)
      IF(AND-D(LM))65,65,80
   65 AN=AN+1.
      KK=1
      IF(LM-N)70,75,70
   70 IK=IK+1
      GOTO60
   75 FS(L)=N
      FC(N)=RC
      GOTO150
   80 IF(KK-1)85,90,90
   85 LM=LM+1
      IF(AND-D(LM))65,65,100
   90 FS(L)=IK
      FC(IK)=RC
      GOTO125
C
C     START OF PHASE III
C
  100 TD=2.*D(IK)
      IN=AN
      IF(TD-D(IN))105,105,110
  105 FS(L)=IK-1
      FC(IN)=RC
      GOTO125
  110 FS(L)=IK
      FC(IK)=RC
C
C     END OF PHASE III
  125 JL=J-L
      IF(JL-1)130,135,135
  130 LN=N
      GOTO140
  135 LN=TS(L1)
  140 AK=0
      LK=IK
      DO145II=LK,LN
      D(II)=AK*D(II)*CINV
  145 AK=AK+1.
  150 CONTINUE
C
C     END OF PHASES II , III
      CC=0
      DO170IJ=1,N
      IF(FC(IJ))165,160,165
  160 FCC(IJ)=CC+D(IJ)
      GOTO170
  165 FCC(IJ)=CC+FC(IJ)
  170 CC=FCC(IJ)
C     PUNCH FINAL RESULTS
      PUNCH3
      PUNCH4
      PUNCH6
      PUNCH7
      PUNCH8
      DO180I=1,N
  180 PUNCH9,I,D(I),PPV(I),CPV(I),TCC(I),FCC(I)
      PUNCH3
      PUNCH11
      GOTO500
    3 FORMAT(80H=========================================================
     X========================)
```

```
    4 FORMAT(26X,11HPART PERIOD,3X,11HCUMULATIVE,3X,9HTENTATIVE,5X,5HFIN
     XAL)
    6 FORMAT(2X,6HPERIOD,6X,6HDEMAND,20X,11HPART PERIOD,4X,10HCUMULATIVE
     X,4X,10HCUMULATIVE)
    7 FORMAT(28X,5HVALUE,10X,5HVALUE,8X,4HCOST,10X,4HCOST)
    8 FORMAT(80H---------------------------------------------------------
     X-----------------------)
    9 FORMAT(4X,I2,9X,I3,8X,F8.2,7X,F8.2,5X,F8.2,6X,F8.2)
   11 FORMAT(/)
      END
```

CASE (1)

PERIOD	DEMAND	PART PERIOD VALUE	CUMULATIVE, PART PERIOD VALUE	TENTATIVE CUMULATIVE COST	FINAL CUMULATIVE COST
1	20	0.00	0.00	50.00	
2	20	20.00	20.00	60.00	
3	25	50.00	70.00	85.00	
4	35	105.00	175.00	135.00	
5	30	30.00	30.00	150.00	
6	10	20.00	50.00	160.00	
7	10	30.00	80.00	175.00	
8	15	60.00	140.00	225.00	

PERIOD	DEMAND	PART PERIOD VALUE	CUMULATIVE, PART PERIOD VALUE	TENTATIVE CUMULATIVE COST	FINAL CUMULATIVE COST
1	20	0.00	0.00	50.00	50.00
2	20	20.00	20.00	60.00	60.00
3	25	50.00	70.00	85.00	85.00
4	35	105.00	175.00	135.00	135.00
5	30	30.00	30.00	150.00	150.00
6	10	20.00	50.00	160.00	160.00
7	10	30.00	80.00	175.00	175.00
8	15	60.00	140.00	225.00	225.00

CASE (2)

PERIOD	DEMAND	PART PERIOD VALUE	CUMULATIVE, PART PERIOD VALUE	TENTATIVE CUMULATIVE COST	FINAL CUMULATIVE COST
1	10	0.00	0.00	100.00	
2	90	90.00	90.00	190.00	
3	10	20.00	110.00	290.00	
4	90	90.00	90.00	380.00	
5	10	20.00	110.00	480.00	
6	90	90.00	90.00	570.00	

PERIOD	DEMAND	PART PERIOD VALUE	CUMULATIVE, PART PERIOD VALUE	TENTATIVE CUMULATIVE COST	FINAL CUMULATIVE COST
1	10	0.00	0.00	100.00	100.00
2	90	90.00	90.00	190.00	190.00
3	10	20.00	110.00	290.00	210.00
4	90	90.00	90.00	380.00	310.00
5	10	20.00	110.00	480.00	320.00
6	90	90.00	90.00	570.00	420.00

CASE (3)

##

PERIOD	DEMAND	PART PERIOD VALUE	CUMULATIVE, PART PERIOD VALUE	TENTATIVE CUMULATIVE COST	FINAL CUMULATIVE COST
1	10	0.00	0.00	100.00	
2	20	20.00	20.00	120.00	
3	40	80.00	100.00	200.00	
4	10	30.00	130.00	300.00	
5	20	20.00	20.00	320.00	
6	40	80.00	100.00	400#00	
7	10	30.00	130.00	500.00	

##

##

PERIOD	DEMAND	PART PERIOD VALUE	CUMULATIVE, PART PERIOD VALUE	TENTATIVE CUMULATIVE COST	FINAL CUMULATIVE COST
1	10	0.00	0.00	100.00	100.00
2	20	20.00	20.00	120.00	120.00
3	40	80.00	100.00	200.00	200.00
4	10	30.00	130.00	300.00	230.00
5	20	20.00	20.00	320.00	330.00
6	40	80.00	100.00	400.00	430.00
7	10	30.00	130.00	500.00	430.00

##