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FORECASTING FOR MANAGEMENT OF SEASONAL GOODS INVENTORIES

by

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INTRODUCTION

Inventories appear on the Balance sheets as an asset. That is among those things of monetary value an enterprise owns. Yet they are viewed by many businessmen as an apparently necessary drain on resources. They tie in capital, occupy space and their keeping and handling costs money. Yet they could be considered as productive of earnings as other types of capital investment.

Magee,(1) in one of his articles describes inventories as follows:

"They serve as the lubrication and springing for a production distribution system, keeping it from burning out or breaking down under external shocks. They help to absorb the effects of errors in forecasting demand, to permit more effective use of facilities and staff in the face of fluctuations in sales, and to isolate one part of the system from the next in order to permit each to work more effectively."

Inventory situations are fundamentally alike, each involving some aspect of cost, service, and usage. The objective in any given situation is to make that set of decisions which will minimize total costs and provide an acceptable or economical level of service at the expected demand or usage rate.

Thus whether it is the case of raw materials, inventory, finished goods inventory, hotels, airlines or railroad reservations, etc., the problem is usually solved on an item basis where it is required that the following information be determined and also continually reviewed for each item:

1. Appropriate costs
2. Expected service level or demand
3. Permissible incidence of stock shortages
4. Forecast of usage
5. Replenishment characteristic e.g. lead time.

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(1) John F. Magee, "Guides to Inventory Policy", Harvard Business Review, May-June, 1956 p 58

In general effective inventory control requires the determination of how much to carry in order to have enough so that demand can be met, but not so much that the cost of inventory becomes excessive relative to demand. Thus the right level of inventory (the optimal inventory), is that which results in minimizing the total of:

1. Cost of acquiring inventory
2. Cost of carrying inventory
3. Cost of demand lost or deferred owing to inventory shortage.

Most of the inventory situations are concerned with studying the maintained inventory that realizes a minimum cost, and establishing order points and order quantities for each inventory item which satisfy those cost and service criteria.

<u>Anticipation Stocks:</u>

There exist situations in which the anticipation of stocks is of more importance than the other factors considered. In these cases forecasting the optimum number of stock to be maintained overshadows the other factors e.g. number and quantity of orders, their location in time, etc.

These could be generalized under those cases where sales are highly seasonal, and the cost of obsolescence or having stock left over at the end of a season is great, and represents a lot of money.

Examples of these situations are the following:

1. Retail merchants, who deal with seasonal style items such as summer dresses, bathing suits, winter coats. ... etc.
2. Manufacturers of highly seasonal goods such as textiles, soft beverages, ... etc.
3. Hotels, airlines and railroads and other similar services which have seasonal swings.

4. Manufacturers of complex equipments such as industrial machinery, some home appliances, automobiles, aircrafts, and components.
5. Manufacturers or merchants running special seasonal deals or sales such as specially labelled packages for certain specified seasons.

and others.

In all these cases, the right forecast of the optimal inventory stock is of paramount importance because if it is too high, serious losses are faced with through write-offs or even destruction of the leftovers after the season. On the other hand, a too low estimate will cause loss of profit that could have been made, besides the loss of customers' good will.

These could be divided into two main categories according to their being influenced by:

1. The "crash" or short-peak season problem, e.g., highly styled seasonal goods.
2. The "seasonal swing" characteristics of sales, e.g., automobiles, furniture, etc.

This paper is only concerned with the crash or short-peak season problems.

The "Crash" Problem:

Deals with the determination of true optimal amount of highly seasonal goods with which the enterprise can expect on the average to break even on the last unit produced or purchased.

So, in the "crash" type of problem, management must balance the risks of not having enough stocks to fill demand and thus losing profit, against the risks of having too much on hand and consequently losing because of obsolescence, write-off or even storage expense until the next selling season.

The classical newsboy problem could go as a good illustration case for the approach to that type of problem, and two-way decisions could be made. But it must be considered as a special case, since the season's length is known and precise; also that he makes his decision very frequently and, therefore, has more chance to arrive at the nearest accurate forecast or determination of his optimum balancing point.

The Classical Newsboy Case:

A newsboy has on the average 10 customers a day who are willing to buy papers costing 5ç each. The newsboy makes a profit of 3c on each paper he sells, and loses 1c on each paper he takes out but fails to sell. It is supposed that he has kept records, and that 40% of the time he can sell at least 10 papers and 20% of the time he can sell at least 12 papers.

If the newsboy does not know how many papers he will actually sell in any given day but every day takes out 10 papers, he has a 40% chance of selling all the papers and making 3ç each, and a 60% chance of not selling all papers and losing 1ç on each not sold. He can expect the tenth paper to produce, on the average overtime, a profit of

$$3ç \times 40\% - 1ç \times 60\% = 0.6ç$$

On the other hand, if he takes 12 papers every night, he can expect the twelfth paper to produce on the average overtime, a loss of

$$3ç \times 20\% - 1ç \times 80\% = -0.2ç$$

It would not, therefore, be worth his while to take out 12 papers. As a matter of fact he would probably make the greatest total profit by taking 11 regularly, since he could expect on the average overtime, to do slightly better than break even on the eleventh paper where

$$3ç \times 30\% - 1ç \times 70\% = 0.2ç$$

Mathematical Formulation:

In most of those problems, where the basic elements are the same - balancing the costs and lost profit opportunities of demand exceeding available stock against the costs and losses of having available unused stock or capacity - the relation between the optimum quantity of stock and the different profits or losses per unit could be arrived at by the use of the expected value method, provided that the distribution density function of order or demand is known.

Example:

An item to be sold during the summer season brings a net profit of m piastres for each unit sold and a net loss of n piastres for each unit left unsold when the season ends. If the distribution of order volume v may be approximated by the density function;

$$f(v) \quad (0 < v < V),$$

and we want to determine the optimum number of units x to be stocked in order to maximize the expected value $\bar{P}$ of the profit P.

Solution:

First put down the two forms of profit functions that exist depending upon whether v is less or greater than x.

Thus we get:

$$P(x) = mv - n(x-v) = (m+n)v - nx \qquad (v < x)$$

$$P(x) = mx \qquad (v > x)$$

Hence the expected value of the profit is

$$\bar{P}(x) = \int_0^x \left[(m+n)v - nx\right] f(v)\,dv + \int_x^V mx\,f(v)\,dv$$

If we rewrite $\int_x^V mx \; f(v) \; dv$ as

$$mx\left[1 - \int_0^x f(v)\,dv\right]$$

the expected value becomes

$$\bar{P}\ (x) = mx + (m+n) \int_0^x (v-x)\ f\ (v)\ dv$$

Differentiating this function with respect to x, we obtain

$$\frac{\delta \bar{P}\ (x)}{\delta x} = m + (m+n)\ (x-x)\ f\ (X) - (m+n) \int_0^x f(v)\ dv$$

$$= m - (m+n)\ F\ (x)$$

Hence, the maximum average profit is reached if x is so chosen that

$$F\ (x) = \frac{m}{m+n} = \frac{\text{unit profit}}{\text{unit profit + unit loss}}$$

If m = 10 P.T. and n = 20 P.T. , then x must be chosen so that the area under f (v) to its left will equal 1/3.

In the case where a company also loses customers' good will besides the loss in sales in the case of lack of several items, this is bigger the greater the number items of that could not be supplied, putting it a constant for simplicity we get

$$\bar{P}\ (x) = \int_0^x \Big[(m+n) - n\ x\Big]\ f\ (v)\ dv + \int_x^V \Big[mx - a\ (v-x)\Big] f\ (v)\ dv$$

where a is the loss to company for each item not available to a customer.

Differentiating this function with respect to x, we obtain

$$F\ (x) = \frac{m+a}{m+a+n}$$

Therefore, for the best strategy add the loss per item due to good will to the margin of profit per item in order to determine an effective margin of profit.

Short-cut Incremental Method:

Schlaifer[2] arrived at the same results as those of the general approach to those problems. He calls his method the short-cut Incremental Method, for locating the last unit to add to the decision quantity. It is based on finding the highest value of j for which the incremental expected profit of stocking is positive. It is assumed that the incremental profit is also positive for all lower values.

Thus, if the cumulative distribution function of the demand is known or preassigned to the different events, the steps of the method are:

1. Determine the values of k_p and k_n by analyzing the incremental cash flows which result from stocking and either selling or failing to sell "one more unit".
2. Compute the "critical ratio". $k_p / (k_p + k_n)$
3. From the values already assigned to $p(Z)$, compute $p(\bar{Z} < j)$ for j = 1, 2, 3, etc., until the last value of j has been found for which $p(\bar{Z} < j)$ is less that the critical ratio.

Thus, in our model where the c.d.f. of demand is normal the value of j could be found from the standard corresponding tables.

This short-cut incremental method is applicable only when the conditional profits k_p and k_n are the same for every unit which might be added.

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(2) Robert schlaifer, "Probability and statistics for Business Decisions" (New York: Mc Grow-Hill 1959) PP. 69 - 76

The Forecasting Method for Management of Seasonal Style-Good Inventories:

The management of seasonal style-goods inventories differs from the management of other inventories in one important characteristic, namely, that the "sudden death" of the highly styled and highly seasonal merchandise is involved. The cost of obsolescence, i.e., of having stock left over at the end of the season, overshadows all other costs of carrying inventories. As for various reasons other than obsolescence, e.g., spoilage, wear and tear, the merchandise cannot be stored for a subsequent season, and has to be disposed of at a lower price. Sometimes even the selling price after the end of a season is so reduced that the out-of-pocket loss per unit to the manufacturers or distributor is greater than the profit per unit, had the item been sold during the regular selling season.

In general, the problem of the case of seasonal style-goods inventories lies in the balancing of the risk of not having stock against the loss of having too much left over.

Herz and Schaffir's Model:

Hertz and Schaffir (3) describe a forecasting method they developed to assist a textile manufacturere in controlling finished stocks of style goods. Their aim was to establish an inventory policy similar to that known for highly seasonal goods which are sold at one price during the season, and disposed of at a lower price at the end of the season.

They balanced the cost and lost profit opportunities of demand exceeding available stock, against those costs and losses of having available unused stock, keeping the prices fixed, while selecting them (if possible) so as to get an appropriate value for the probability of selling their marginal unit of stock.

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(3) Operations Research Vol. VIII, No. 1 (January- February, 1960) pp 45 - 52.

They then adopted the normal curve as a predictor to cumulative sales, and used the sales data as basis for the determination of the duration and timing of the selling seasons. Together with the line ratio concept for forecasting total sales for the season at different probability levels, they tried to trace the demand distribution normal curve.

They then made the necessary corrections for the estimated relations they used, mostly on emperical basis.

<u>The Model</u>

- Hertz and Schaffir based their model on the following assumptions:-
- The price at which the item will be offered for sale is established in advance, and remains fairly stable throughout the selling season.
- The sales of a line j, follow a growth curve pattern which can be approximated by a cumulative normal curve of mean u_j and variance v_j^2.
- The timing of the season is known (u and v given).
- The ending dates of the season can be fixed very accurately since in that particular case they are tied to retail selling seasons that culminate at Easter and Christmas.
- A "season" for a line is defined as beginning or "opening" in the week in which sales for the line to date reach 5 percent of season total, and ending (or "closing") in the week when sales to date reach 95 percent.

Categorization into lines:

Fabrics are grouped into lines that are defined as including all items manufactured for sale in a single distribution channel, in the same price range, for the same functional end use. Thus, men's flannel shirting material sold in the range of 1.50 to 2.00 per yard would be considered as one line. They would be of different colour but of the same particular design; all are offered for sale jointly (as a sample, collection or display) and exhibit the same seasonality pattern.

Determination of the sales pattern's parameters:

Since the season for a line is from the week it reaches 5 percent of seasonal total, to the week where sales to date reach 95 percent of season total.

Also, since the sales of a line follow the normal curve, the season is thus fixed as starting in week (u - 2v) of the lines sales normal curve, reaching its peak sales in week u and ending in week (u + 2v).

Thus if the season opens at time 0 when 5 percent of total expected sales have been realized then

$$\frac{u - 0}{v} = w\,(0.05) = 1.645$$

hence

$$v = \frac{u}{1.645}$$

and if the season's length is known, u would be its mid point and v its standard deviation could be calculated from that formula.

Calculation of net expected return or (cost):

For each unit of an item placed in inventory at a given time, e_{ikw}, its net expected return or cost is calculated from the formula:

$$e_{ikw} = P_{ikw}\ C_i + (1 - P_{ikw})\ C'_i$$

where

P_{ikw} is the probability that the k th unit of item i will be sold by the end of the regular selling season. This probability is reevaluated at each week (w) in which this evaluation is made.

C_i = the unit profit for the ith item if sold during the regular selling season and found from

$$C_i = P_i - r_i$$

(P_i = selling price, r_i variable cost)

C_i' is the unit profit or loss based upon the reduced price P_i' and r_i . To maximize the expected profit, additional units will be added to stock as long as e_{ikw} is positive i.e. $e_{ikw} \geqslant 0$

or $$0 \leqslant P_{ikw}\ C_i + (1 - P_{ikw})\ C_i'$$

from which $$-C_i'\ (1 - P_{ikw}) \leqslant C_i\ P_{ikw}$$

or $$-C_i' + C_i'\ P_{ikw} \leqslant C_i\ P_{ikw}$$

hence $$-C_i' \leqslant (C_i - C_i')\ P_{ikw}$$

and $$P_{ikw} \geqslant \frac{C_i'}{C_i' - C_i}$$

This could be expressed in terms as

$$\text{Probability of selling k or more units} = \frac{\text{Profit (or loss) per unit sold after end of season}}{\text{Profit (or loss per unit sold after end of season} + \text{Lost profit per unit of Unsatisfied Demand During Season}}$$

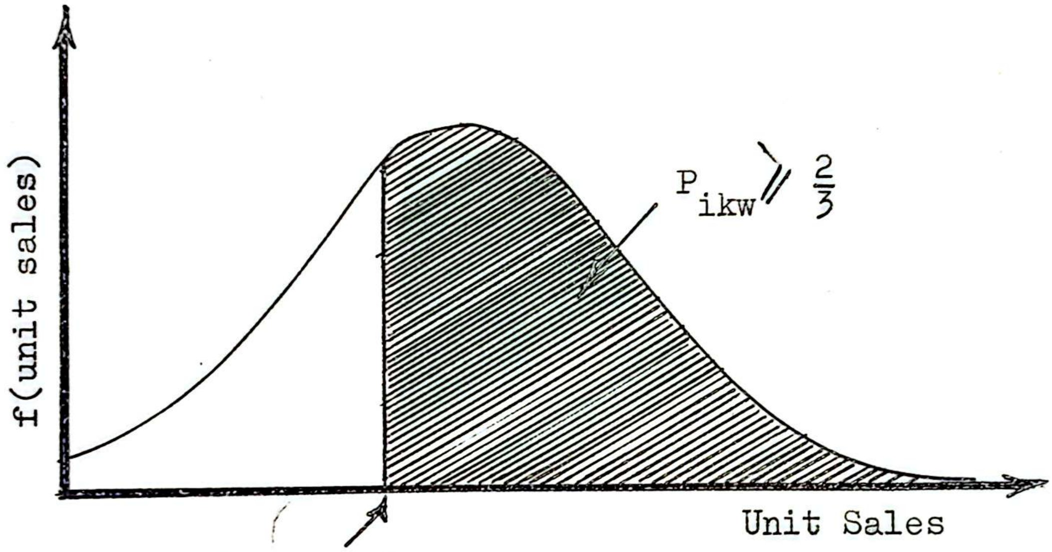
Example (1)

Putting loss of having a unit excess stock = 1.00 and loss in profit resulting from being short in unit \$ .50

then
$$P_{ikw} = \frac{-\$\,1.00}{-\$\,1.00 - \$\,.50} = \frac{2}{3}$$

Therefore stock should be set at a level which will allow sales to exceed stock two times out of three. That means that the best policy to adopt, is <u>to set inventory at a level that will permit total demand to be greater than the amount stocked twice as frequently as total demand is less than the amount stocked.</u>

This could be represented graphically as follows:



Optimum No. of units to be placed in stock.

Example: (2)

For a 10 weeks season, u would be 5 and $v = \frac{5 - 0}{1.645} = 3.04$

From the normal growth curve, the expected values for each week during seasons of various lengths can be obtained.

Calculation of forecasting ratio g_w^* :-

Given u and v, values for the forecasting ratio g_w^* (fraction of cumulated sales up to week w to total season sales) are obtained from tables of the normal distribution

$$g_w^* = \frac{1}{v\sqrt{2\pi}} \int_{\infty}^{w} \exp\cdot \left[-(w-u)^2/\ 2v \right] dt$$

The week w must be located in time with respect to the beginning and end of the season.

Example:

To obtain cumulative sales at the end of the 6th week in a 10 week season

$$g_6^* = \frac{6-5}{3.04} = 0.33 \text{ from tables} = 63 \text{ per cent of total sales.}$$

Obtaining $\hat{S}_i$ the forecast of the total expected sales:

A forecast $\hat{S}_i$, of total sales for the season for a specified item is obtained by dividing sales to date S_{iw} by the appropriate forecasting ratio:

$$\hat{S}_i = \frac{S_{iw}}{g_w^*}$$

Thus in our example if we put the actual cumulative sales to date equal to 6,400 units at the end of the 6th week

estimated total sales would be

$$\frac{6400}{0.63} = 10,160 \text{ units}$$

Determination of probability limits for the deviation from the assumed sales pattern:

These forecasts of the total sales for the season have an important limitation. They fail to make allowance for the fact that cumulative sales at any given time will not represent the exact proportion of total sales indicated by the normal growth curve. And there will be a distribution of "actual" cumulative sales about expected cumulative sales.

Therefore the forecast ratio has to be adjusted so that the chance of having either too little or too much stock at the end of the season will be such that cost is minimized.

In other words, the forecast ratio used must be chosen so that the expected frequency with which actual sales will exceed stock will be equal to the probability that the k th item will be sold by the end of the season.

Thus assuming that g_{iw} has a normal distribution around its mean and σ is the standard deviation of this distribution, then we first calculate $\hat{g}_{iw}$ in the adjusted forecast ratio from the equation:

$$\hat{g}_{iw} = g^*_w + h\,\sigma_w$$

σ_w was determined from historical sales data for different lines and with different lengths of seasons.

For a line of m items, it is given by

$$\sigma_w^{\,2} = \frac{1}{m} \sum_{i=1}^{i=m} (g_{iw} - g^*_w)^2$$

Further the forecast ratio used must be such that the expected frequency with which actual sales will exceed stock will be equal to P_{ikw} i.e. The total season sales S_i will be equal to or greater than forecast with probability α

where $\alpha = P\left[S_i \geqslant \hat{S}_{iw}\right]$

This figure is used to determine the number of standard deviation units to be added or subtracted from the forecast ratio obtained for the week from the normal growth curve

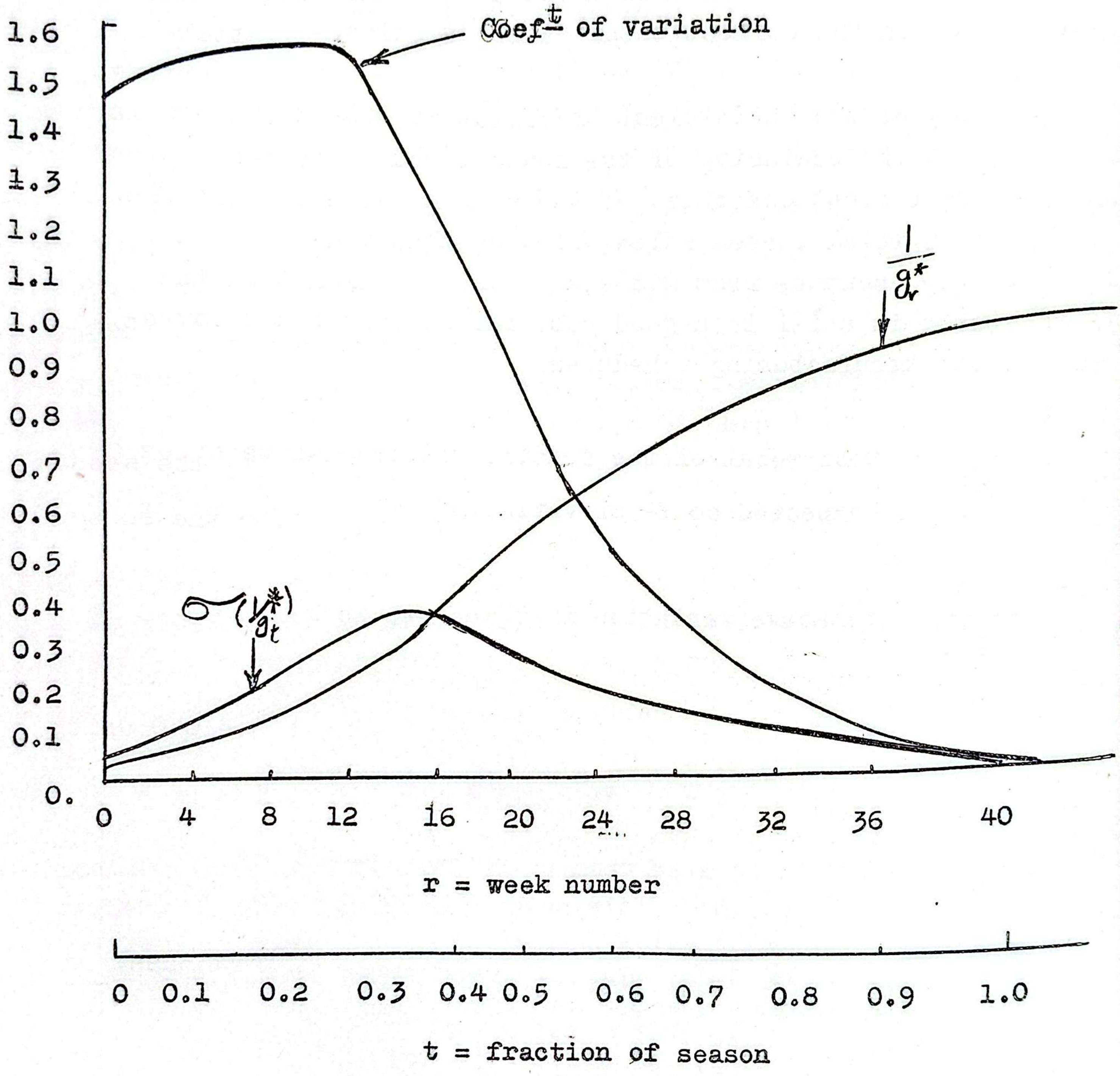
The adjusted ratio is then divided into the total cumulative sales to obtain the correct total season's sales.

A study of the coefficient of variation showed that it is very high at the beginning of the season, and decreases rapidly after about one third of the season passed. Thus the estimates of total season sales based on that model become increasingly accurate from mid season on, at which time the manufacturer is still in a good position to adjust his inventories, and manufacturing schedules.

$\frac{1}{g^{*}_{w}}$ = reciprocal of the fraction of cumulative sales

expected coef$^{\underline{t}}$ of variation

$\sigma\left(\frac{1}{g^{*}_{t}}\right)$ = relative variation from week to week


Coef$^{\underline{t}}$ of variation
$\frac{1}{g_r^*}$
$\sigma(1/g_t^*)$
1.6
1.5
1.4
1.3
1.2
1.1
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.
0 4 8 12 16 20 24 28 32 36 40
r = week number
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0
t = fraction of season

The coefficient of variation curve showing the relationship of σ_w to w in the shown chart could also be read as the relation between σ_w and the n the week of the season if the scale (fraction of the season) is used.

The Authors remarked that as expected, high volume items exhibit significantly less variation than small sellers. Also that they did not think that the use of other growth pattern curves than the normal would yield consistently reduced variances from actual sales data. Yet they could not predict the nature of the deviation of some of the individual lines from the normal curve pattern.

Example:*

Assume a forecast of sales is required for an item at the end of the 6th week and that the season is fixed by 10 weeks, given the following data:

1. $1.5 is the unit profit of an item sold during the season

   $0.5 is the unit profit of an item sold after the season

2. the distribution of forecast ratios is approximately normal, and the standard deviation of the forecast ratio in week 6 is $\sigma_6 = 0.15$

3. Cumulative sales made through the 6th week are 640 units from 1. we calculate such that

$$P_{ikw} = \frac{-0.50}{-0.50 - 1.50} = 0.25$$

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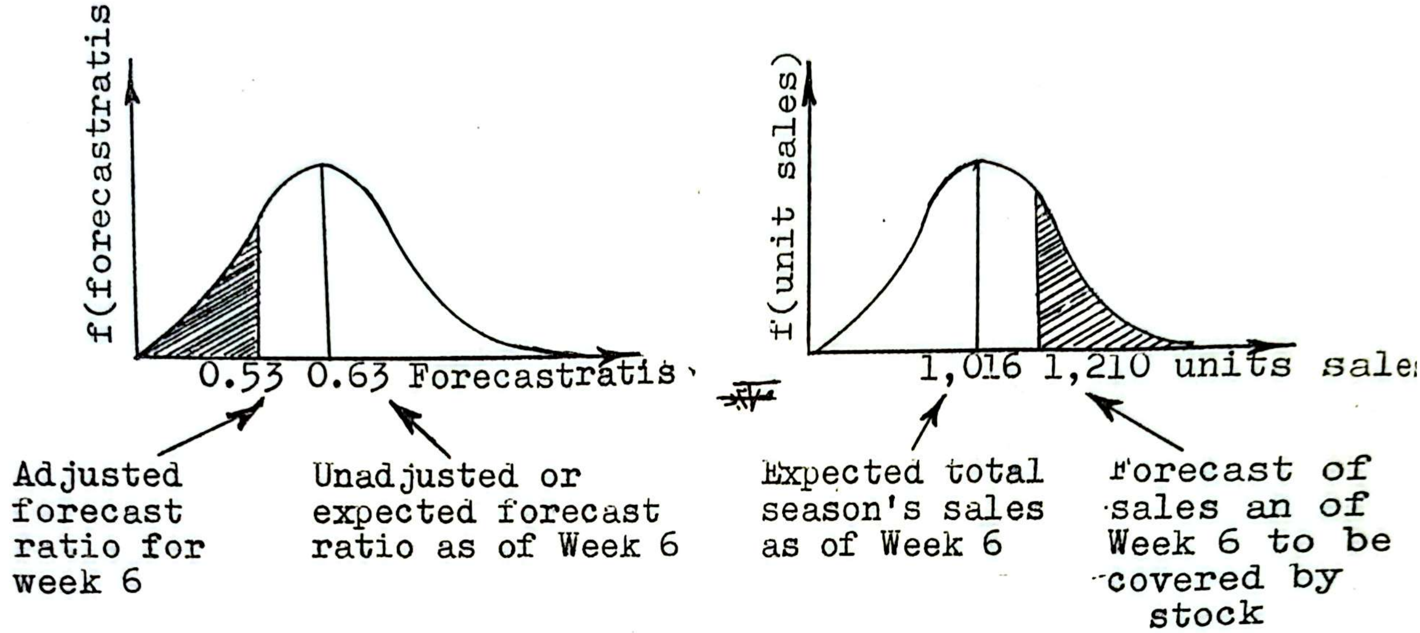
*Mathematical Models and Methods in Marketing by Frank Bass and others (Irwin 1961)

Expected sales for the total season was calculated before by dividing the cumulative sales to date by the unadjusted forecast ratio (0.63) and was found to be

$$\frac{640}{0.63} = 1{,}016 \text{ units}$$



To obtain the adjusted forecast ratio for the 6th week

$$0.25 < g_w = \frac{g_{i6}(0.25) - 0.63}{0.15}$$

from tables $$-0.674 = \frac{g_{i6}(0.25) - 0.63}{0.15}$$

or $$g_{i6}(0.25) = 0.63 - 0.15 \times 0.674$$
$$= 0.53$$

hence we obtain the adjusted forecast of sales as of the 6th week from

$$\frac{640}{0.53} = 1{,}210$$

The graph shows that to allow demand to exceed stocks not more than one time in four, the true forecast ratio must not exceed the adjusted forecast ratio more than three times out of four.

Adjustment for the error in the estimate of the Length of the Season:

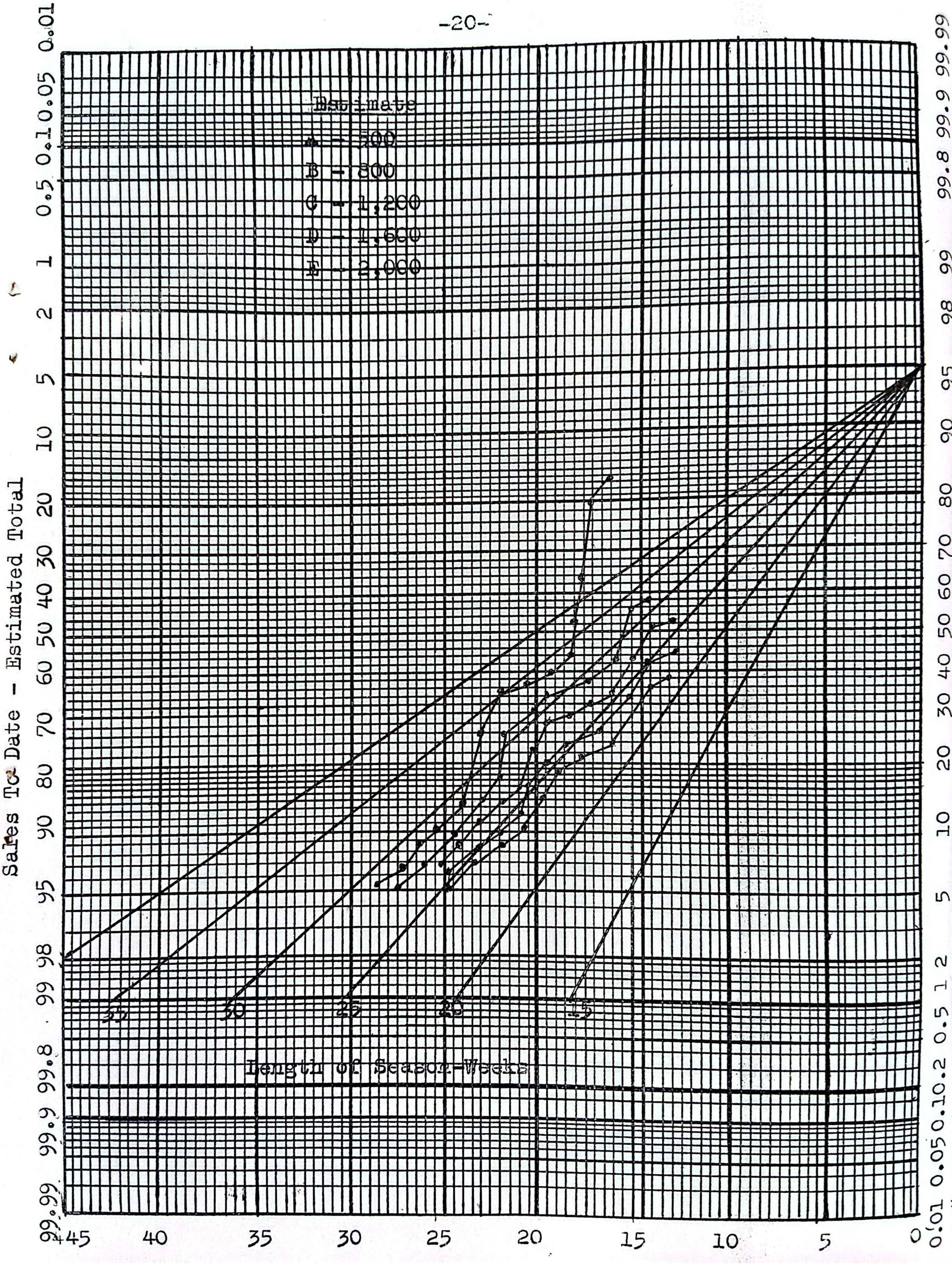
All the forecasts and probabilities we got up till now were based upon the assumption that the timing of the season is known. The unadjusted forecast ratio on which we practically based all our procedures were obtained from an assumed cumulative normal curve, and the parameters of this normal curve were easily calculated once the length of the season was known.

In practice however it is impossible to determine with certainty when 5 per cent of the season's sales have been realized. The uncertainty with which the stage of the season any line has reached will exist during the entire season but decreases as the season advances.

Although in this particular case the ending dates of the seasons are tied to retail selling seasons that culminate at Easter and Christmas, yet the starting point of any season may occur over a wide period of time.

Determination of the length of the season:

Knowing the cumulative sales to date for each of the preceding weeks and the end point of the season, the Authors developed a method for the determination of the stage of the season any given line has reached. With the aim to obtain n the length of the season, and the total sales of the season, they first drew the standard straight lines representing the ratio of sales to date divided by total season sales, based on the assumption that sales will follow a normal growth curve reaching 95 per cent of total season sales in a known week. They assumed their season to vary between 15 weeks and 40 weeks. They then calculated the weekly ratio of sales to date to various assumed levels of the total season sales. The series of ratios that most nearly fits the normal growth curve is the one obtained by using the most nearly correct forecast of total season's sales.


Estimate
A - 500
B - 800
C - 1,200
D - 1,600
E - 2,000
Sales To Date - Estimated Total
Length of Season-Weeks
35
30
25
20
15
45
40
35
30
25
20
15
10
5
0

From the figure it is obvious that the best estimate has the corresponding plotted points most closely the straight line approaching 95 per cent point in the end-of-season week. It is also obvious that too low an estimate will cause the points to diverge upwards (representing a higher ratio) and too high an estimate will make them diverge downwards.

In the figure the best estimate appears to be at about 1,200 units. The corresponding estimate of the length of the season is 27 weeks.

The authors state that in the actual application of the system, a tabular version of this method was provided.

<u>Combination of the variances of the Sales Forecasts:</u>

To account for the error in the estimation of the net expected return on a specific unit to be put into inventory, a certain number of variances has to be added.

Since the error is caused mainly due to the uncertainty relating to the estimated length of season, the variance for g^*_w the "forecasting ratio" estimator must be obtained. The rest of the error is due to random fluctuations that are not related to the uncertain length of season and a separate variance for that must be obtained.

Then the total variance attributable to both random occurrances is obtained by adding the two variances (the distribution of the sum of independent variables has a variance equal to the sum of the variances of the corresponding industrial variables).

Thus σ^2_{nw} the variance of the unadjusted sales forecast due to error in length of season n in week w is obtained. This is then used for the determination of $\sigma^2_{n^*w}$ the variance of the unadjusted forecast ratio due exclusively to uncertainly in the true length of season. Then adding $\sigma^2_{n^*w}$ to the previously used for the determination of $\sigma^2_{n^*w}$ the variance of the unadjusted forecast ratio due exclusively to uncertainty in the true length of season. Then adding $\sigma^2_{n^*w}$ to the previously used σ_w^2 gives the total variance of the sales forecast.

a. <u>Determination of σ_{nw}^2 the variance of the unadjusted sales forecast due to error in length of season n in week w:</u>

The authors pointed out that they got it empirically from the deviations between actual and estimated lengths of reason for various lines. With the end of season date given, their method has given an estimate of the length of season that is within 2-3 weeks during the last 18-20 weeks of the season for 90 per cent of the cases studied.

Also they found that the value of σ_{nw}^2 during the critical middle third of the season is on the order of 1 - 1.5 weeks or 4-6 per cent of the length of an average season of 25 weeks.

b. <u>Determination of $\sigma^2_{n^*w}$ the variance of the unadjusted forecast ratio due exclusively to uncertainty in the true length of a season:</u>

We know that $\hat{S}_{iw} = \dfrac{S_{iw}}{g_w^*}$ S_{iw} = cumulative sales through week w

this could be written as $\hat{S}_{iw}$ = forecast of total sales made up to week w

$S_{iw} = g_w^* \ \hat{S}_{iw}$ g_w^* = unadjusted forecast ratio for week w

In any week $\hat{S}_{iw}$ can be treated as a random variable. By selecting a particular g_w^*, the value of this random variable is determined by the current value of S_{iw}.

By a the theorem in probability theory, that states "the variance of a constant times a random variable is equal to the square of the constant times the variance of the random variable", σ_{n^*w} is calculated as follows:

$$\sigma_{n^*w} = g_w^{*2} \; \sigma_{nw}^{2}$$

where σ_{nw}^{2} is the variance of the variable $\hat{S}_{iw}$ the weeks unadjusted forecast.

c. As mentioned before the total variance σ_{w}^{*2} is obtained from the summation of the two variances as follows:

$$\sigma_{w}^{*2} = \sigma_{w}^{2} + \sigma_{n^*w}^{2}$$

The standard deviation σ_w^* is then used in place of σ_w in the previous equation

$$\hat{g}_{iw\alpha} = g_w^* + h_\alpha \sigma_w$$

Combining it with an appropriate value of P_{ikw}, adjusted estimates of the total sales may be obtained which lead to optimum stock levels.

Conclusion:

This model is very interesting, and the method used in it for sales forecasting is different than the chemical one.

The authors use the balancing formula's of the cost and exceeds of both cases when inventory exceeds sales and when sales exceeds inventory, to arrive at the probability of the optimum situation. Yet instead of applying this probability with a normal cumulative curve of demand as the others do, they assume cumulative sales to follow the normal curve, although without any evidence to it being so. They then take the forecast ratio for a line obtained in any week during the season as an estimator of of the performance of the items in that line. Thus the estimated total sales for an item may be obtained as seen, with the corrections then made for the error of both the uncertainty of length of reason (which they assumed to be known) and the error due to other characteristics of the operation.

As for the pair of variances the authors got empirically and the question is whether there would be a way to overcome this, since the model is supposed to be used to help management to forecast total selling, and those corrections are made from past experience.

Further the application of such models in industry, to meet the special problem of highly seasonal goods, management organization should be set up so as to provide positively for coordination among long-range inventory planning and production scheduling. For sometimes from the scanty experience gained in early season selling, enough information can be developed so that estimates of total season sales and resulting production plans can be adusted.

E.Z.