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Development of a Dual Economy

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<u>Development of a Dual Economy</u> *

Most of the developing economies are dual in character. One segment of them is quite comparable to the Western capitalist economies, where economic principles that have been evolved are more or less applicable. We shall designate this segment as the industrial sector. The other segment of these economies is such that economic principles have limited applicability as the productive activity rests largely on traditions. We shall call this sector the traditional sector. The main distinguishing feature of these two sectors, on which the following discussion rests, is that while the output attributable to the last worker in the industrial sector is equal to the wage-rate, it is not necessarily so in the traditional sector. The latter is a fair description of the situation since most of the productive activity in the traditional sector is conducted in form of household enterprises, where any equality between the inputed earnings and the contribution of a working member of the family is only accidental. In the industrial sector, however, the assumption of the equality between the wage-rate and the marginal productivity of labour is quite reasonable. We shall assume that the average wage rate is the same in both the sectors; the familiar assumption that all the wages are consumed is implied. The role of rate of interest is suppressed on the assumption that a developing economy in its early stages is very short of capital and has abundant profit opportunities, so that all savings are automatically invested.

These assumptions are common to those used by Ranis and Fei in their interesting articles[1] which have served as the starting point for the following study. We have tried to relax some other assumptions used by

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"1" Gustav Ranis and J.C.H. Fei: "A Theory of Economic Development" Americ Economic Review, 1961 ________: Innovation, Capital Accumulation and Economic Development" American Economic Review 1963.

* This memo. contains the model as such Mr. M. Hosny in association witl Mr. Khairy is engaged in finding numerical solutions . I am gratef to Dr. Salah Hamid for allowing the Use of computation facilities and for his encouraging attitude in general.

the co-authors which we think are too restrictive and have tried to answer some other questions which were not raised by them, but which we think are very pertinent and important. The two crucial assumptions[2)] made by Ranis and Fei, which are mainly responsible for the elegant and simple result they derive, are that the marginal physical productivity changes at a constant rate as employment in agricultural sector varies, i.e., MPP curve is a falling straight line and that'an increase in agricultral productivity shifts the entire TPP (total physical productivity) curve "upward" proportionaly', i.e., 'the new TPP curve is obtained by multiplying the initial TPP curve by a constant'. The major shortcoming of the models of the coauthors is that they discuss the disposition of the agricultural output, but ignore that of the industrial output. The process of capital formation which is central to any dynamic growth theory is conspicuously absent. The opportunity of determining the marginal productivity of factors in the two sectors through the reallocation of growing labour and capital in the balanced way with which they are mainly concerned is simply forgotten. The result is that their approach can at best be described as partial comparative static. Their second article does little to alter this view[3)] Since it simply demonstrates that variations in the rate of labour employment over what is employed in proportion to the growth of capital is due to technological changes and innovation. Our aim is to relax the assumptions and to remove as far as possible the shortcomings stated above.

In section II, we give the main balanced growth model and its solution under the assumption of a given rate of growth of employment in the industrial sector. Section III deals with optimal allocation factors of production. Section IV takes up the problem of the commercialisation point, viz., the time point when the marginal productivity in both the traditional as well as the industrial sector equal the wage rate. Section V studies the effect of the inflow of foreign capital on the commercialization point. In section V,

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"2" Ranis and Fei:"The Theory of Economic Development", American Economic 1961, pp. 456-47.

"3" cf. Am. Ec. Rev. 1963.

we envisage a situation in which the traditional sector does not only produce items of direct consumption, but also raw materials to be used in the industrial sector.[4]

The model assumes full-utilisation of all capital. The productive activity in both sectors is assumed to be characterised by a homogeneous production function of degree one. The demand pattern does not change. Discontinuity in the production function due to innovation etc. does not occur.

II

The Model

The model constitutes of 13 equations

$$P = P_1 + P_2 \tag{1}$$

$$P_1 = P_1 (L_1 , K_{o1}) \tag{2}$$

$$P_2 = P_2 (L_2 , K_{o2}) \tag{3}$$

$$P_1 = C_1 L + \dot{K}_1 \tag{4}$$

$$P_2 = C_2 L + \dot{K}_2 \tag{5}$$

$$K_o = K_o^o + \int_1^t K_1 \, dt + \int_1^t \dot{K}_2 . \, dt \tag{6}$$

$$K_o = K_{o1} + K_{o2} \tag{7}$$

$$L = L_1 + L_2 \tag{8}$$

"4" As it has not been possible to find a mathematical solution to the model and work at numerical solutions is still under way on the IBM 1620 of the Operations Research Centre, section III and the succeeding ones will be taken up later, may be, through use of same variational calculus.

$$L_o = L_o^o \, e^{nt} \tag{9}$$

$$L_2 = L_2^o \, e^{n_2 t} \tag{10}$$

$$W = \gamma \frac{\partial P_2}{\partial L_2}, \quad \gamma \leq 1 \tag{11}$$

$$W = C_1 + C_2 \tag{12}$$

$$C_2 = C_2^o \, e^{mt} \tag{13}$$

Equation (1): Total output in the economy, p. is equal to the sum of the outputs in the traditional sector, p_1, and that of the industrial sector, p_2. We shall denote the symbols and the parameters partaining to the traditional sector with subscript 1 and the industrial sector with subscript 2.

Equation (2): The production possibilities in the traditional sector where L_1 is the amount of labour employed and k_{o1} is the amount of capital utilized (capital includes land) is represented by the Cobb-Douglas production function,

$$P_1 = u_1 \, L_1^{a_1} \, K_{o1}^{b_1} \tag{2}$$

both the elasticities a_1 , b_1 are greater than zero.

Equation (3): The production function is given here also by the Cobb-Douglas type:

$$p_2 = u_2 \, L_2^{a_2} \, K_{o2}^{b_2} \tag{3}$$

where the elasticities a_2, b_2 are both positive and L_2 is the amount of labour employed in the industrial sector and K_{o2} is the amount of capital utilized.

Equation (4): Out of the traditional sector production a quantity related to the total population as C_1 L is consumed and the remaining part $\dot{K}_1$ is invested

Equation (5): Here also, the output of the industrial sector, p_2 is partly consumed at the average rate of C_2 and the rest, $\dot{K}_2$, goes to capital formation.

Equation (6): Total capital stock is equal to the initial capital plus the sum of additions to it from both sectors.

Equation (7): The total capital stock is then fully utilized in both sectors.

Equation (8): The sum of L_1, L_2 is the total labour force, L.

Equation (9): The total labour force is growing with a rate n, with an initial value L^o.

Equation(10): The labour absorption in the industrial sector has a rate of growth n_2. The employment in the initial period is L_2^o.

Equation(11): The average wage rate W, which equals the average per capita consumption is supposed to be the same in the two sectors.

Equation(12): The average wage rate is the sum of the average consumption for worker of the traditional output and industrial output.

Equation(13): The trend of the average percapita consumption of the industrial output is given an exponential shape with the rate m.

We have 13 unknowns; namely:

$P = p^t$, $p_1 = p_1^t$, $p_2 = p_2^t$,

$L = L^t$, $L_1 = L_1^t$, $L_2 = L_2^t$

$K_o = K_o^t$, $K_{o1} = K_{o1}^t$, $K_{o2} = K_{o2}^t$, $\dot{K}_1 = \dot{K}_1^t$, $\dot{K}_2 = \dot{K}_2^t$

$W = W^t$, $C_2 = C_2^t$

III

Some Modifications

If we have to deal with the case where investment is initiated only in the industrial sector i.e. $\dot{K}_1 =$ zero, considering C_1 as constant, we have, ($\gamma = 1$ throughout this section)

$$C_1 = \frac{P_1}{L}$$

$$\text{i.e. } W = \frac{P_1}{L} + C_2 = u_2 a_2 L_2^{-(1-a_2)} K_{o2}^{b_2}$$

also we have

$$K_{o2} = K - K_o - \left(\frac{P_1}{u_1 \ L_1^{a_1}}\right)^{1/b_1}$$

$$K_{o2} = \left(\frac{P_2}{u_2 \ L_2^{a_2}}\right)^{1/b_2} = \left(\frac{\dot{K}_o + C_2 \ L}{u_2 \ \ L_2^{a_2}}\right)^{1/b_2}$$

This gives the path of K_o^t in this case by the differential equation.

$$K_o = \left(\frac{\dot{K}_o + C_2 L}{u_2 \ \ L_2^{a_2}}\right)^{1/b_2} + \left\{ \frac{L}{u_1 \ (L-L_2)^{a_1}} - \left(\frac{a_2}{L_2} - (K_o + C_2 L) - C_2\right) \right\}^{1/b}$$

where $\quad L = L^o \ e^{nt} \ , \ L_2 = L_2^o \ e^{n_2 t} \ , \ C_2 = C_2^o \ e^{mt}$

Another modification may be that the traditional sector provi the industrial one with a technically determined portion as raw materia i.e. replacing $\dot{K}_1$ in our original model by $\theta \ p_2$(where $\theta$is a given techr parameter). We can then derive, considering C_1 as a constant

$$P_1 = C_1 L + \theta p_2$$

$$\therefore \quad W = \frac{P_1 - \theta p_2}{L} + C_2 = u_2 a_2 L_2^{-(1-u_2)} K_{o2}^{b_2}$$

also
$$K_{o2} = K_o - \left(\frac{P_1}{u_1 L_1^{a_1}}\right)^{1/b_1}$$

$$K_{o2} = \left(\frac{P_2}{u_2 L_2^{a_2}}\right)^{1/b_2} = \left(\frac{\dot{K}_o + C_2 L}{u_2 L_2^{a_2}}\right)^{1/b_2}$$

and this gives the differential equation:

$$K_o = \left(\frac{\dot{K}_o + C_2 L}{u_2 L_2^{a_2}}\right)^{1/b_2} + \left[\frac{L}{u_1 (L-L_2)^{a_1}} \left\{ \left(\frac{\theta}{L} + \frac{a_2}{L_2}\right)(K_o + C_2 L) - C_2 \right\}\right]^{1/b_1}$$

where

$$L = L^o e^{nt}, \quad L_2 = L_2^o e^{n_2 t}, \quad C_2 = C_2^o e^{mt}$$

It's a well known phenomenon that efficiency of production increase year after year due to technological progress and growing experience.

One simple may to represent technical progress is to multiply the function by an exponential factor. This can be allowed for by writting $u_1 = u_1^o e^{v_1 t}$, $u_2 = u_2^o e^{v_2 t}$ where v_1, v_2 are the annual rates of growth of efficiency in production and can be taken in some cases equal.