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Analytical Study for Expenditure On Food in the Private Sector

By

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ANALYTICAL STUDY FOR EXPENDITURE ON FOOD
IN THE PRIVATE SECTOR

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1. Introduction

No doubt that food is the most important sector of expenditure for the individual in the U.A.R., where 55% of his income is spent. Also the average expenditure per individual on food stuff increases from 1016 gms in 1951/52 to 1261 gms in 1963/64. For this reason it is of interest to study the expenditure on food as a whole and on its different groups separately for rural and urban regions.

In this paper relations between the total annual expenditure and the expenditure on food stuffs are obtained also the expenditure elasticities are discussed.

If z denotes the average expenditure on the different groups of food.

x denotes the average total annual expenditure, it is found that the function $z = a x^2 + bx + c$ is the best that fits most of groups of food stuffs.

For food stuffs added together, cereals and starches and dry beams, the best fitted function is of the form $\frac{1}{z} = a + b/x$ which is linear in $1/z$ and $1/x$.

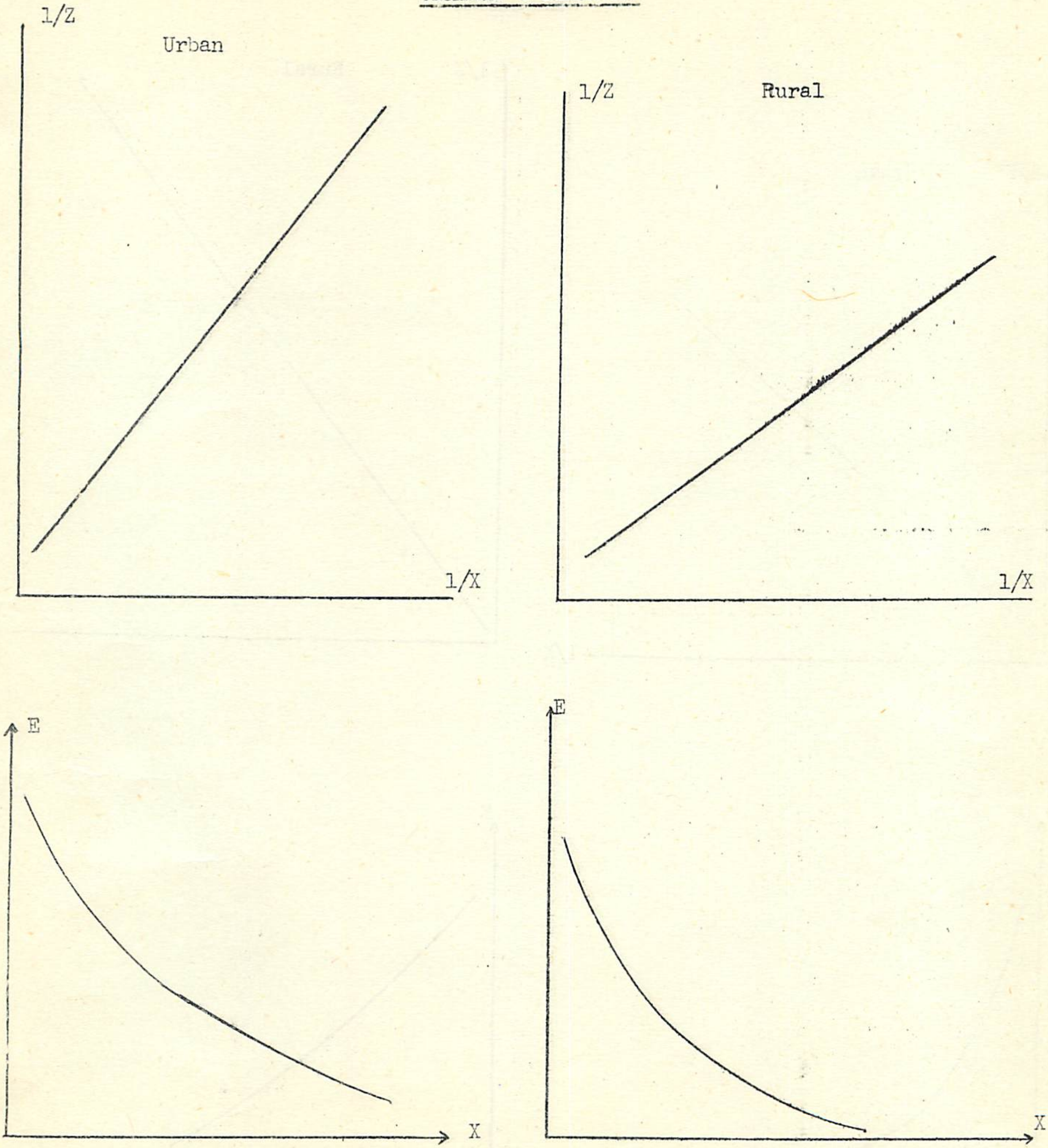
2. Results

The fitted functions $z = f(x)$ and the coefficients of elasticities (E) at the average total expenditure for urban and rural règions are summerized in the following table:

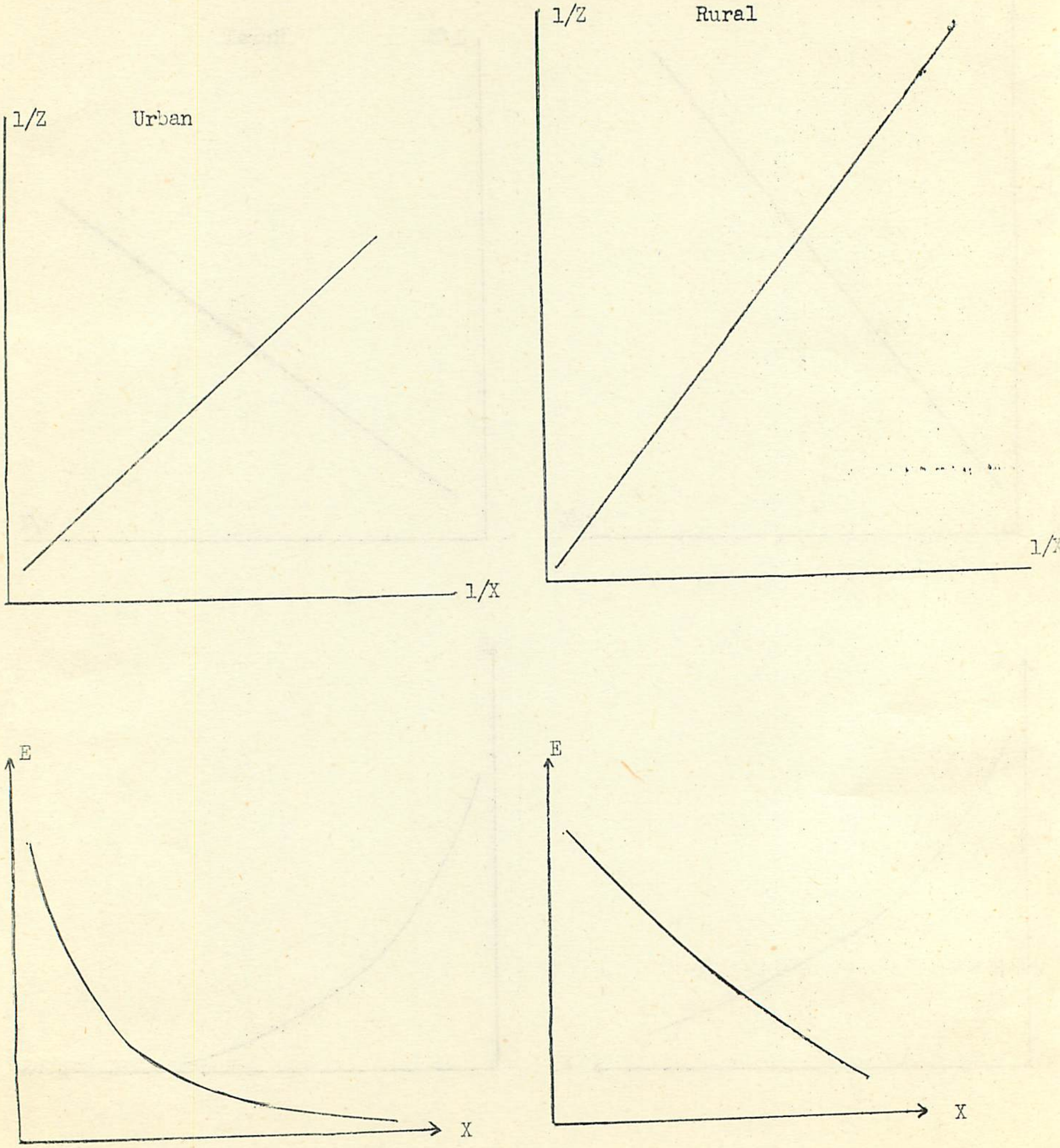
Commodities		$z = f(x)$	E
Cereals & Starches	U	$z = \dfrac{x}{1.30 + .08x}$	0.18
	R	$z = \dfrac{x}{1.83 + .04x}$	0.52
Dry beams	U	$z = \dfrac{x}{22.53 + .73x}$	0.31
	R	$z = \dfrac{x}{32.59 + .08x}$	0.92
Meat, Fish & Eggs	U	$z = 1.33 + .14x - .0002\ x^2$	1.007
	R	$z = .57 + .14x - .0001\ x^2$	1.09
Oils and fats	U	$z = .54 - .01x - .00002x^2$	0.50
	R	$z = -.02 + .02x - .00007x^2$	0.87
Milk & Milk Product	U	$z = -.81 + .09x - .0001x^2$	1.04
	R	$z = .53 + .11x - .0003x^2$	1.05
Vegetables	U	$z = .18 + .04x - .00006x^2$	0.82
	R	$z = -.09 + .04x - .0001x^2$	0.89
Fruits	U	$z = 0.91 + .05x - .00005x^2$	1.30
	R	$x = 0.25 + .03x + .00002x^2$	1.53

Commodities		$z = f(x)$	E
Sugar & Sugar food	U	$z = 0.32 + .03x - .00005x^2$	0.76
	R	$z = 0.26 + 0.03x - 0.00004x^2$	0.83
Other food stuffs	U	$z = .19 + 0.01x + 0.0000005x^2$	0.48
	R	$z = 0.12 + 0.02x - 0.00007x^2$	1.05
All food stuffs	U	$z = x/(1.35 + .0099x)$	0.67
	R	$z = x/(1.29 + .0077x)$	0.82

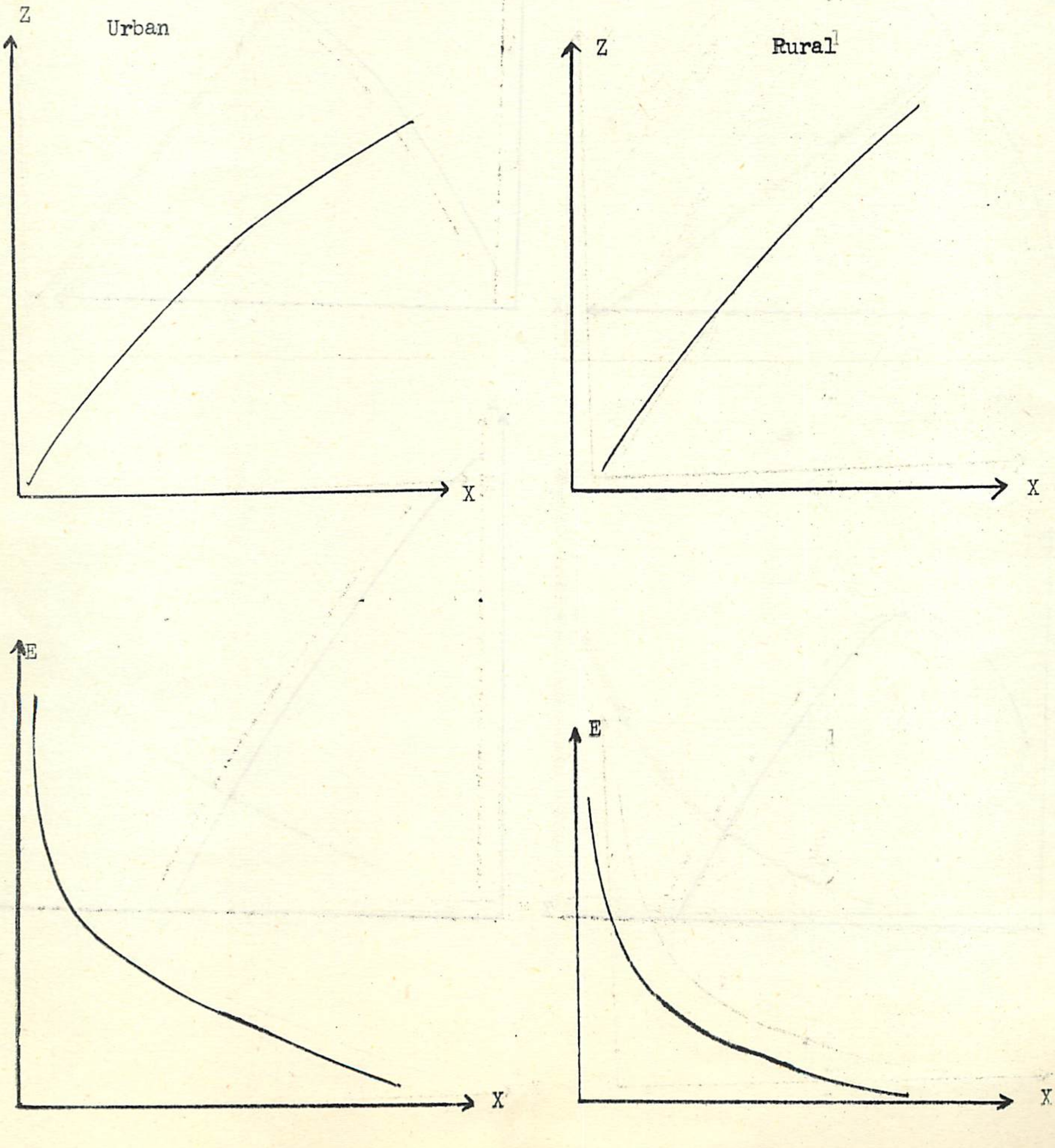
CEREALS & STARCHES



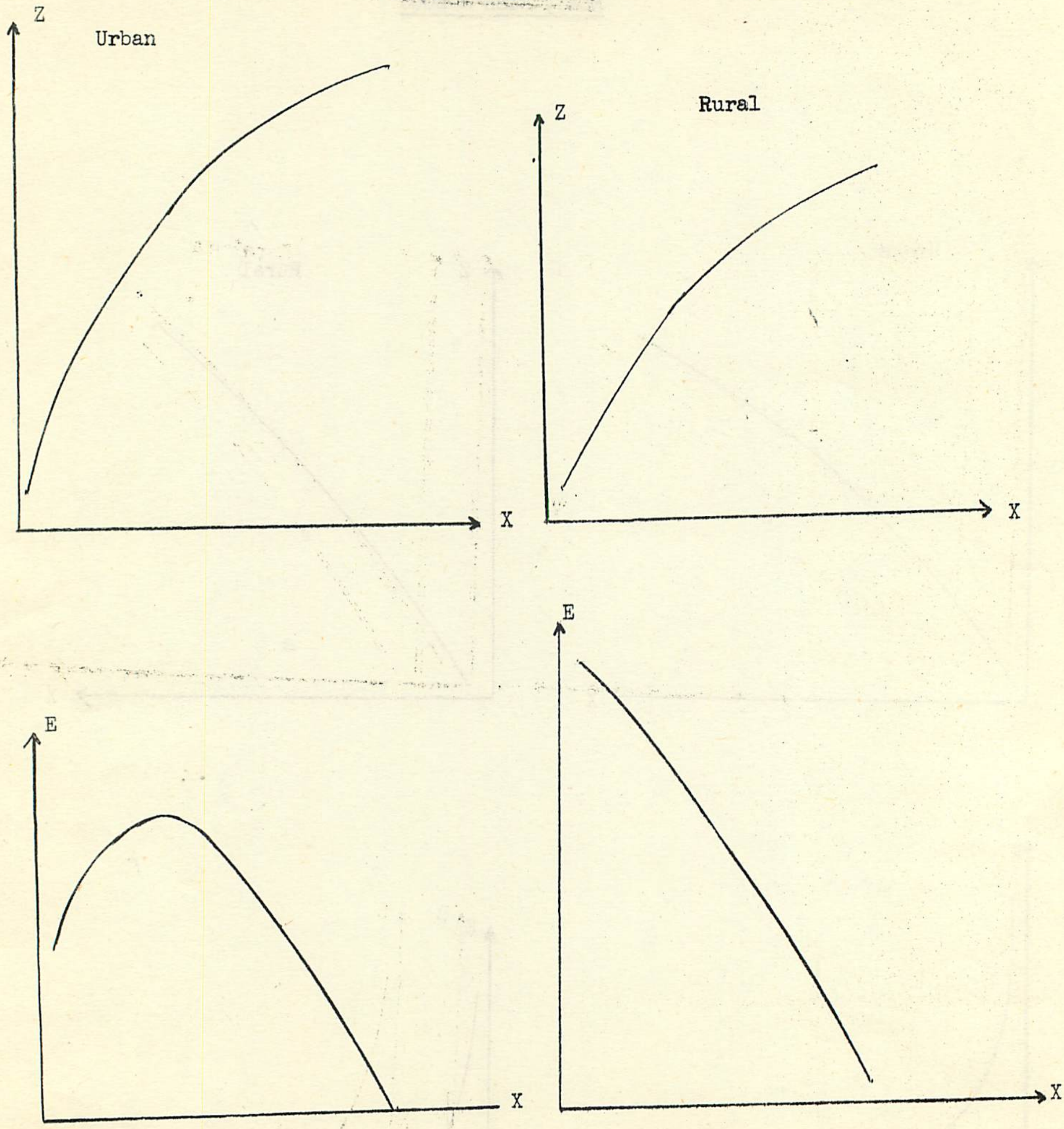
DRY BEANS



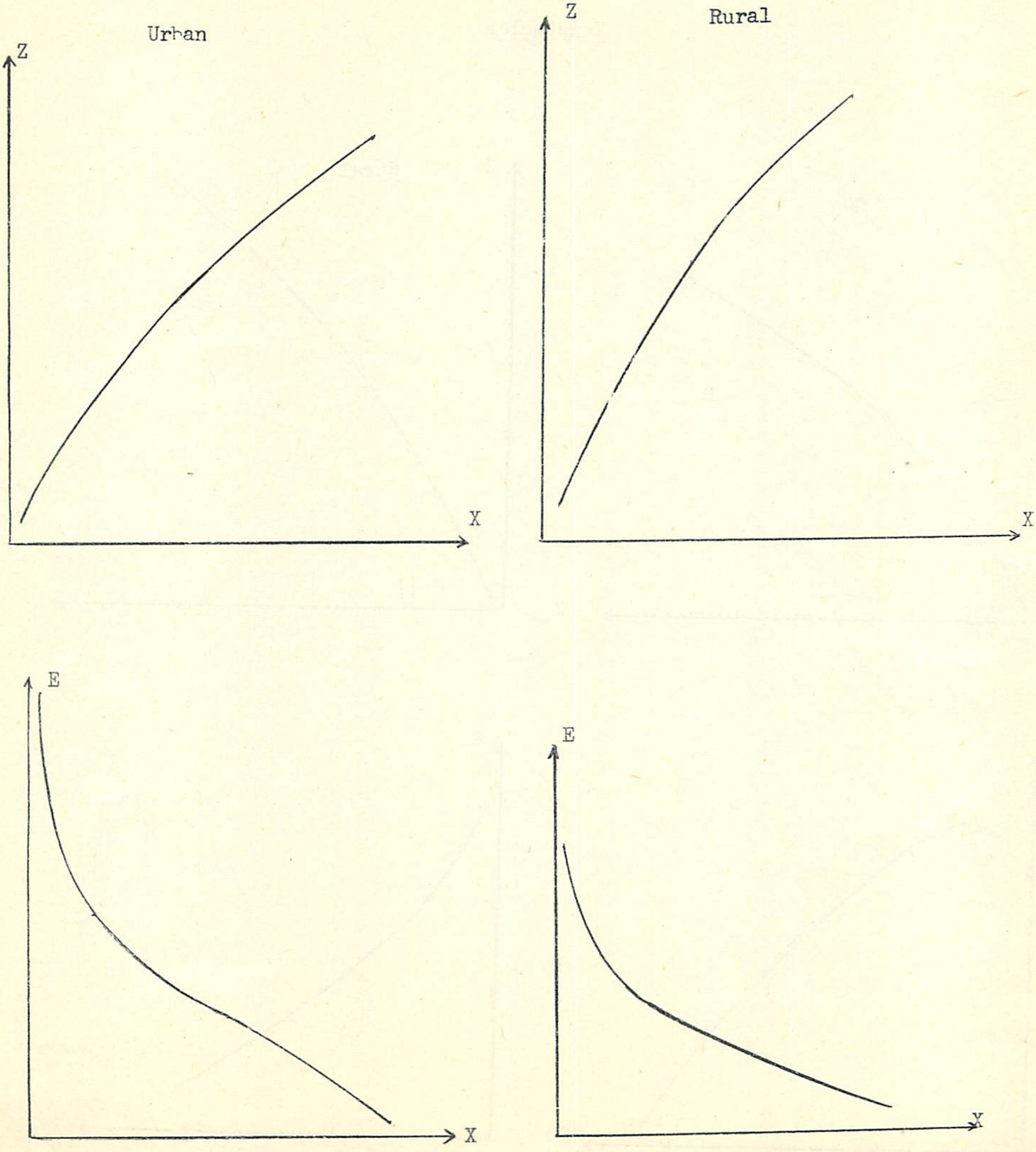
MEAT, FISH & EGGS



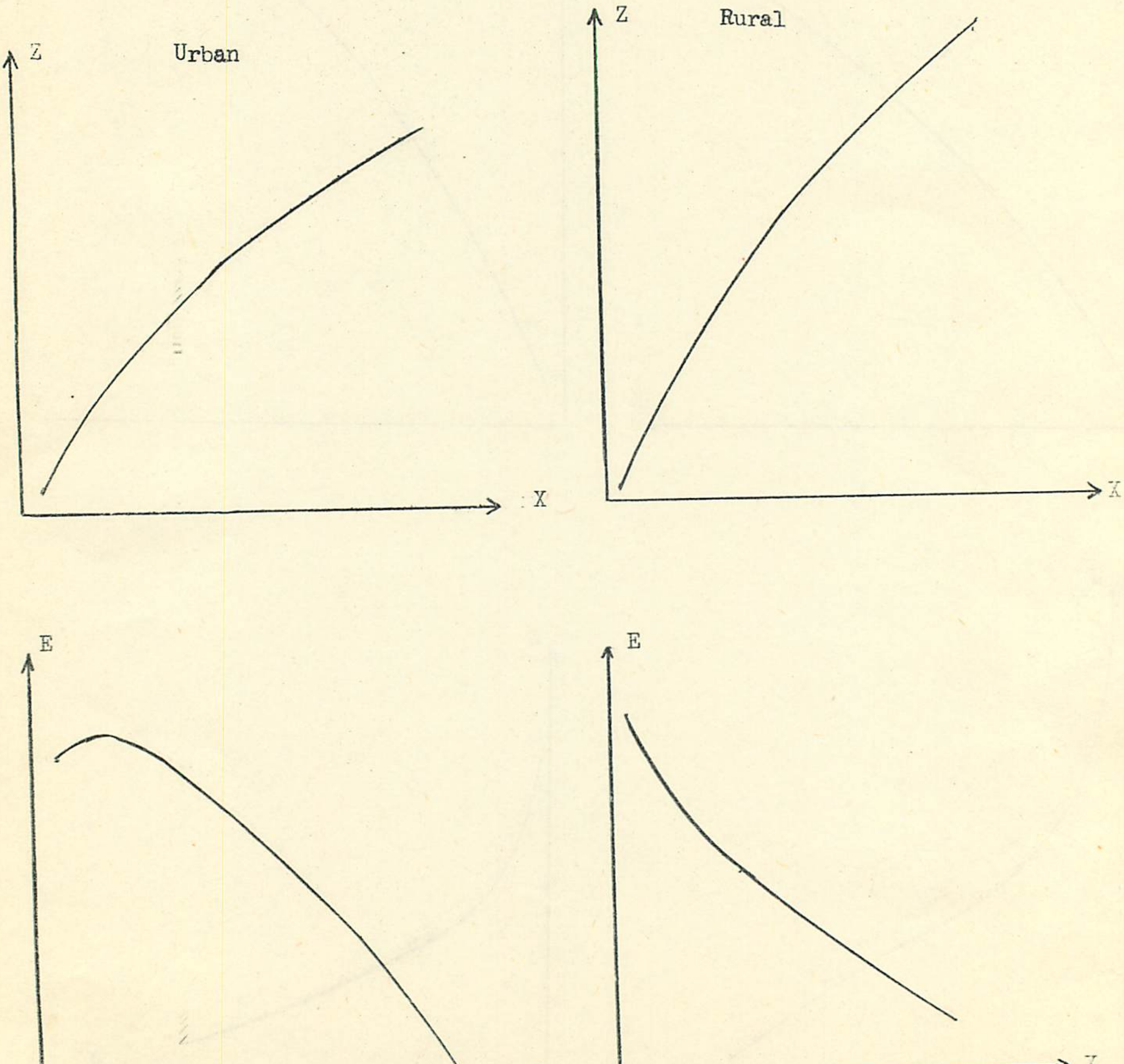
OILS & FATS

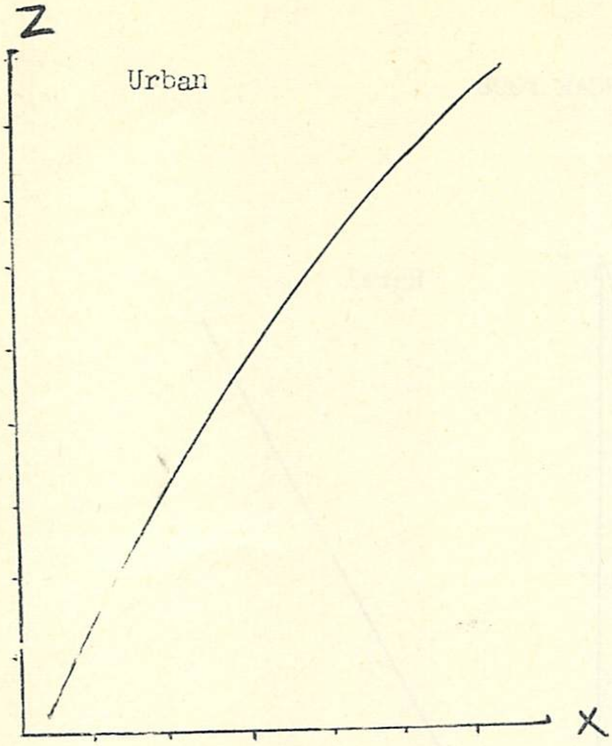


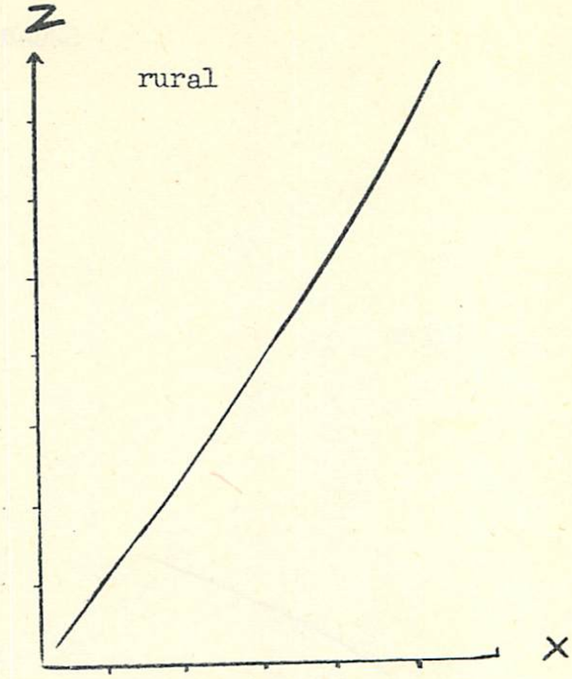
MILK & MILK PRODUCTS



Vegetables



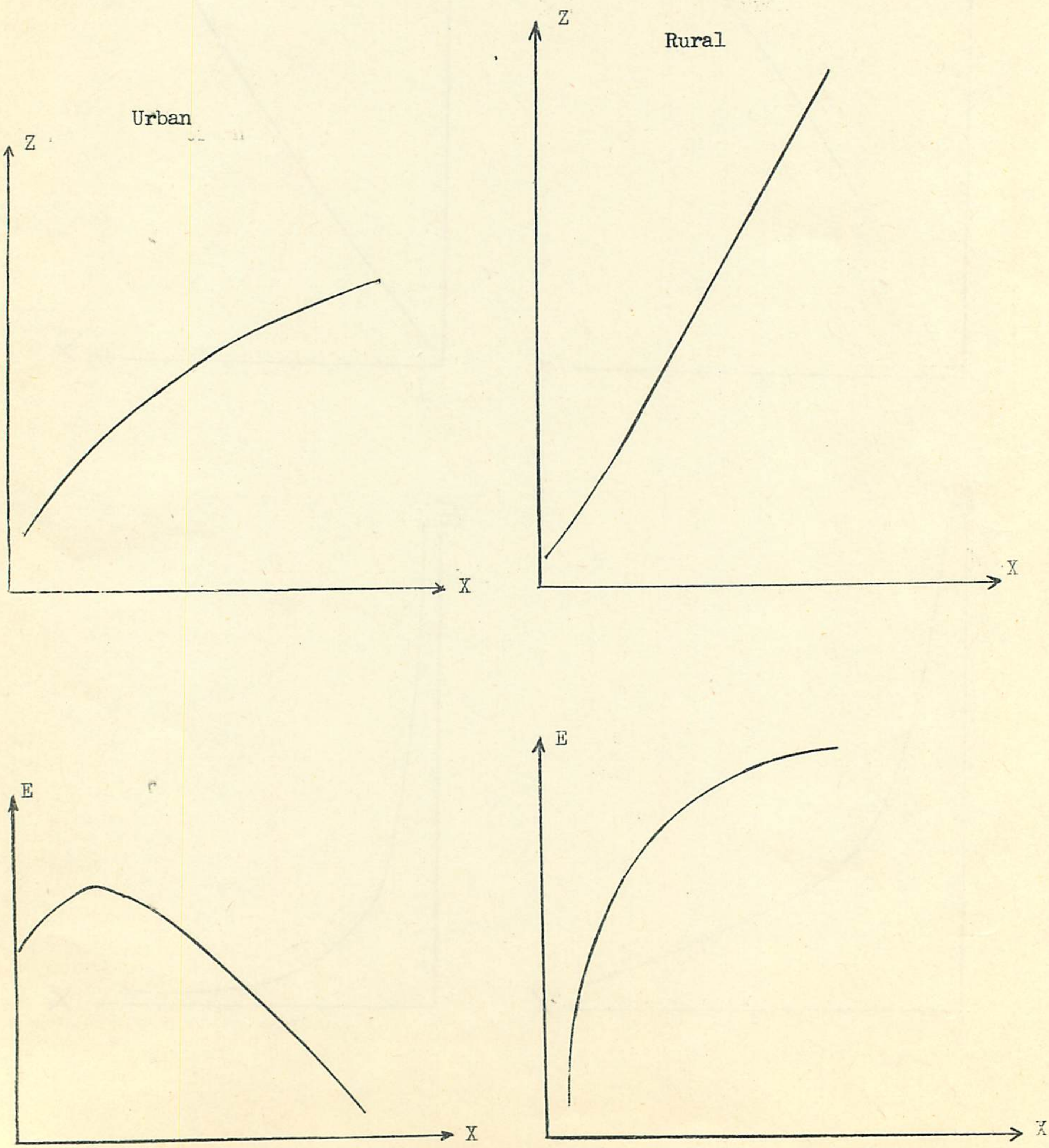

Z
Urban
X


Z
rural
X

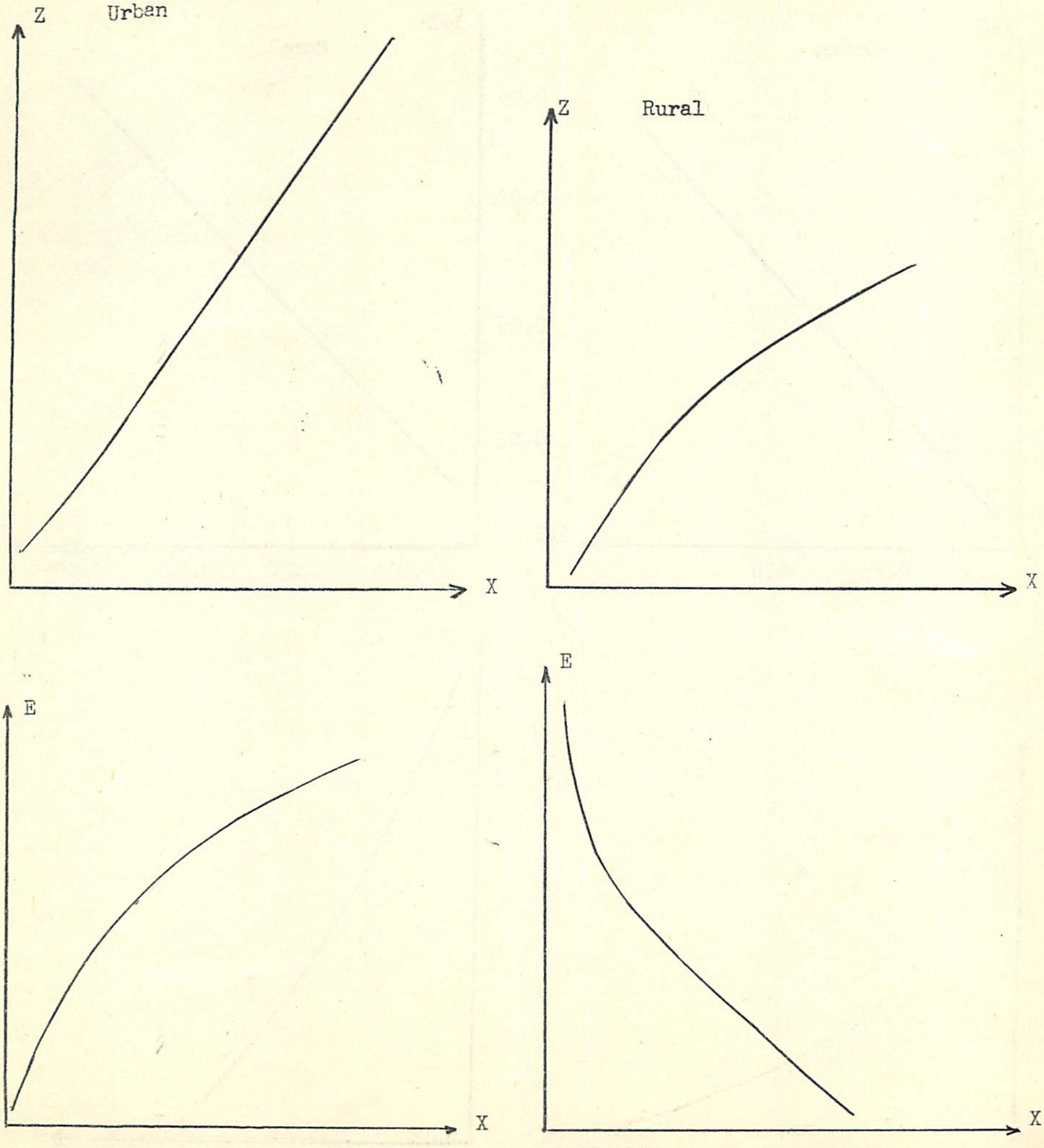

E
X


E
X

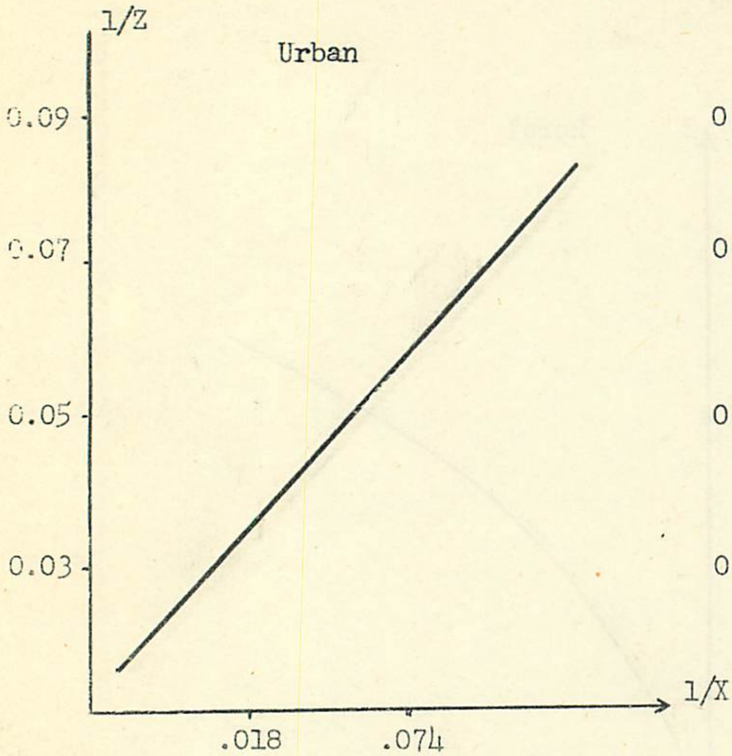
SUGAR & SUGAR FOOD

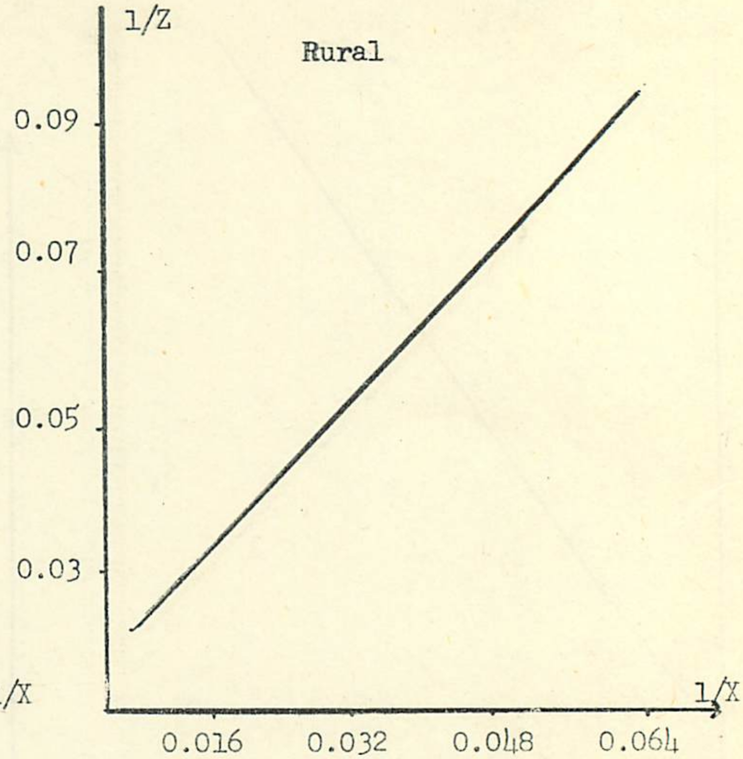


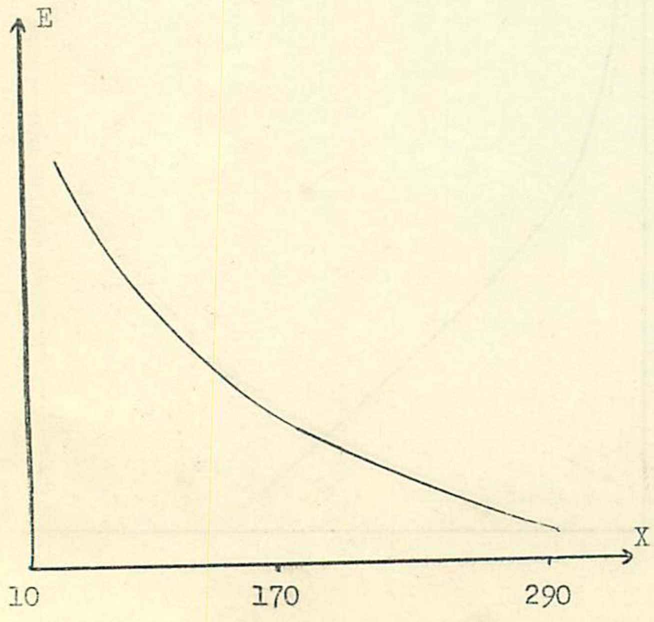
OTHER FOOD STUFFS

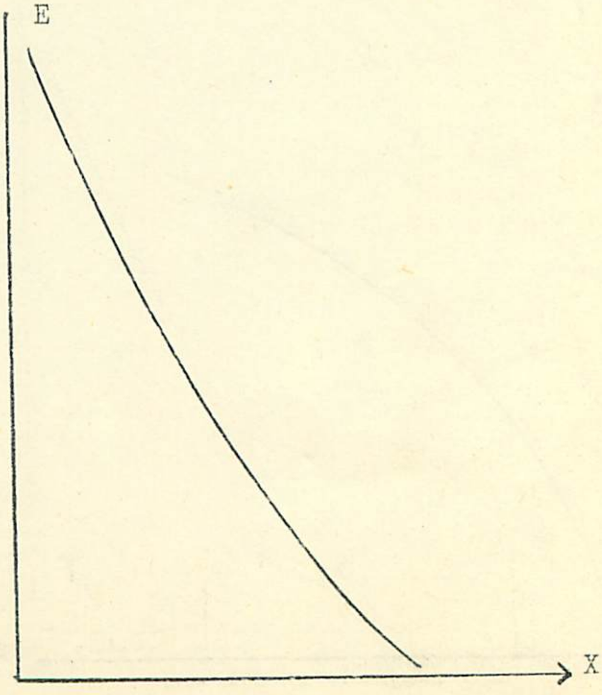


ALL FOOD STUFFS









Appendix

a) Fitting a straight Line

$$z_i = a + b\,(x_i - \bar{x}) \qquad (1)$$

Using the least square method we get

$$\left.\begin{aligned} a &= \bar{z} \\ b &= \sum_{i=1}^{n} (x_i - \bar{x})(z_i - \bar{z}) / \sum (x_i - \bar{x})^2 \end{aligned}\right) \qquad (2)$$

Let the theoritical medol be of the form

$$z_i = \alpha + \beta\,(x_i - \bar{x}) + e_i \qquad (3)$$

where e_i are independent random errors with

$$\mathcal{E}(e_i) = o \quad \& \ \mathrm{Var}\,(e_i) = \sigma^2 \qquad (4)$$

Then

$$\mathcal{E}(a) = \alpha \quad , \ \mathrm{Var}\,(a) = \sigma^2/n \qquad (5)$$

$$\mathcal{E}(b) = \beta \quad , \ \mathrm{Var}\,(b) = \sigma^2/\sum (x_i - \bar{x})^2 \qquad (6)$$

The summary regression analysis table is given as follows:

Source of Variation	Sum of squares	Degree of freedom	Expected value for the mean sum of squares
Due to regression	$b^2 \sum_i (x_i - \bar{x})^2$	1	$\sigma^2 + \beta^2 \sum_i (x_i - \bar{x})^2$
Residual	$\sum_i (z_i - z_i)^2$	n-2	σ^2
Total	$\sum_i (z_i - \bar{z})^2$	n-1	

Assuming that e_i are also normally distributed, then the ratio: Mean sum of square due to regression / Mean sum of squares of residual as $F_{1,n-2}$ and thus used as a criterion for testing the null hypothesis $\beta = 0$ against $\beta \neq 0$

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b- Fitting a parabolic function

$$z_i = a + b_1 x_i + b_2 x_i^2 \qquad (1)$$

This can be rewritten as

$$z_i = b_o T_o (x_i) + b_1 T_1(x_i) + b_2 T_2(x_i) \qquad (2)$$

Where $T_o (x_i)$, $T_1(x_i)$, $T_2 (x_i)$ are orthogonal polynomials of degree o,1,2 respectavely & defined as

$$\left.\begin{aligned} T_o(x_i) &= c_{oo} \\ T_i (x_i) &= c_{11} x + c_{10} \\ T_2 (x_i) &= c_{22} x^2 + c_{21} x + c_{2o} \end{aligned}\right) \qquad (3)$$

Taking $c_{oo} = c_{11} = c_{22} = 1$ and making use of the orthogonlity property

$$T_r(x_i) T_s(x_i) = o \quad , r \neq s \qquad (4)$$

we get:

$$\left.\begin{aligned} c_{o1} &= - \bar{x} \\ c_{21} &= \frac{m'_3 - m'_1 m'_2}{m'_2 - m'^2_1} \\ c_{20} &= \frac{m'^2_2 - m'_1 m'_3}{m'_2 - m'^2_1} \end{aligned}\right) \qquad (5)$$

Where m'_1, m'_2, m'_3 are the moments around the origin.

Now applying the least square method we get

$$\left.\begin{aligned} b_o &= \sum_i z_i T_o (x_i) / \sum_i T_o^2 (x_i) \\ b_1 &= \sum_i z_i T_1 (x_i) / \sum_i T_1^2 (x_i) \\ b_2 &= \sum_i z_i T_2 (x_i) / \sum_i T_2^2 (x_i) \end{aligned}\right) \qquad (6)$$

Let the theoritical model be

$$z_i = \beta_o T_o (x_i) + \beta_1 T_1 (x_i) + \beta_2 T_2 (x_i) + e_i \qquad (7)$$

Where e_i are independent random errors with $\mathcal{E}(x_i) = o$ and Var $(x_i) = \sigma^2$. Then it can be shown that

$$
\left.\begin{aligned}
(b_o) &= \beta_o \quad , \mu_2 (b_o) = \sigma^2/n \\
(b_1) &= \beta_1 \quad , \mu_2 (b_1) = \sigma^2/\sum_i T_1^{\,2} (x_i) \\
(b_2) &= \beta_2 \quad , \mu_2 (b_2) = \sigma^2/\sum_i T_2^{\,2} (x_i)
\end{aligned}\right) \qquad (8)
$$

The regression analysis table can be summerized as follows:

Source of Variation	Sum of squares	Degree of freedom	Expected value of the mean sum of squares
Regression of 2nd order	$b_2^2 \sum_i T_2^2 (x_i)$	1	$\sigma^2 + \beta_2^2 \sum_i T_2^2 (x_i)$
Regression of 1st order	$b_1^2 \sum_i T_1^2 (x_i)$	1	$\sigma^2 + \beta_1^2 \sum_i T_1^2 (x_i)$
Residual	$\sum_i (z_i - z_i)^2$	n-3	σ^2
Total	$\sum_i (z_i - \bar{z})^2$	n-1	

Assuming that e_i are also normally distributed then the ratio: Mean sum of squares due to regression/M.Sum of squares of residual is distributed as $F_{1,n-3}$ and then used as a criterion for testing the null hypothesis $\beta_r = o$ against $\beta_r \neq o$, $(r = 1,2)$

c) Coefficient of Elasticity

Given the function $z_i = f(x_i)$, then the coefficient of elasticity E is defined as

$$E = \frac{x}{z}\frac{dz}{dx} = \frac{x f'(x)}{f(x)} \tag{1}$$

If $\frac{1}{z} = a + \frac{b}{x}$

i.e $z = \frac{x}{b + ax}$ (2)

Then $E = \frac{b}{b+ax}$

Also, if $z = a + b_1 x + b_2 x^2$, then

$$E = \frac{x(b_1 + 2b_2 x)}{a + b_1 x + b_2 x^2} \tag{3}$$

PROGRAM (1)

FOR LINEAR REGRESSION

```
      DIMENSIØN x(20), Y(20), Z(20), Z1(20), Z2(20)
100   READ1, N
  1   FØRMAT (I2)
      READ2, (x(I), I = 1,N)
      READ2, (Y (I), I = 1,N)
  2   FØRMAT(8F10.5)
      READ3, (F(I), I = 1, N)
  3   FØRMAT (10F5.0)
      S1 = 0.
      DØ 4 I = 1,N
  4   S1 = S1 + F(I)
200   S2 = 0.
      S3 = 0.
      DØ 5 I =1,N
      H1 = F(I) * x(I)
      S2 = S2 + H 1
  5   S3 = S3 + H1 * x(I)
      S4 = 0.
      S5 = 0.
      DØ 6 I = 1, N
      H2 = F(I) * Y (I)
      S4 = S4 + H2
  6   S5 = S5 + H2 * Y (I)
      S6 = 0.
      DØ7  I = 1,N
  7   S6 = S6 + F(I) * x(I) * Y (I)
      SSY = S4 * S4 / S1
      SST = S5 - SSY
      SSx = S2 * S2 / S1
      E = S3 - SSX
      SXY = S6 - S2 * S4 / S1
      B = SXY/E
      SSREG = B * B * E
      SSR = SST - SSREG
      SSRM = SSR/(S1 - 2.)
      SGMA = SQRTF (SSRM)
      FSEG = 1.96 * SGMA
      R = SSREG/SSRM
      IS1 = S1
      K1 = 1
```

```
      K2 = IS1 - 2
      S3 = IS1 - 1
      A = (S4 - B* S2)/S1
      PUNCH8,A,B
 8    FØRMAT (5X, 2HA =, F10. 5,5X, 2HB =, F10.5)
      IF (A) 11, 9, 9
 9    PUNCH 10,B,A
 10   FØRMAT (5X, 3HY ^=, F10.5, 4H  X  +, F10.5)
      GØ  TØ  13
 11   PUNCH 12, B, A
 12   FØRMAT(5X, 2HY=, F10.5, 1HX, F10.5)
 13   PUNCH 14
 14   FØRMAT(1X/28X, 25HREGRESSIØN   ANALYSIS   TABLE)
      PUNCH 16
 16   FORMAT (5X, 62H . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . .
     X . . . . . . . . . .)
      PUNCH 17
 17   FØRMAT(6X,19HSØURCE  ØF  VARIATIØN,8X, 5HS.SQ.,8X,4HD.F.,8X,7HM.S.SQ.
     X )
      PUNCH 16
      PUNCH 18, SSREG, K1, SSREG
 18   FØRMAT (6X,17HDUE  TØ  REGRESSION, 6X,E14.4,4X,I5,E14.4)
      PUNCH 19, SSR, K2, SSRM
 19   FØRMAT (6X,8HRESIDUAL, 15X, E14.4, 4X, I5, 4X, E14.4)
      PUNCH 16
      PUNCH 20, SST, K3
 20   FØRMAT(6X, 5HTØTAL , 18X, E14.4, 4X, I5) PUNCH
      PUNCH 16
      PUNCH 21,R
 21   FORMAT (1X/8X, 3HR  =, E14.4)
      PUNCH 22
 22   FORMAT (6X,1HX,15X,1HY,11X,9H UPPER  LIM,6X, 9HLOWER LIM /)
      DØ28  I = 1,N
      Z(I) = B * X(I) + A
      Z1(I) = Z(I) + FSEG
      Z2(I) = Z(I) - FSEG
      IF(Z1 (I) - Y (I) ) 24, 23, 23
 23   IF (Y(I) - Z2(I) ) 24, 26, 26
 24   PUNCH 25, X(I), Y(I), Z1(I), Z2(I)
 25   FØRMAT (2X, F10.5, 5X, F10.5, 5X, F10.5, 5X, F10.5, 5X, 1HW)
      GØTO 28
 26   PUNCH 27, X(I), Y(I), Z1(I), Z2(I)
 27   FØRMAT (2X, F10.5, 5X, F10.5, 5X, F10.5,5X, F10.5, 1HR)
 28   CØNTINUE
      GØTØ 100
      END
```

PROGRAM (2)

FOR QUADRATIC REGRESSION

```
      DIMENSIØN X(50), Y(50), F(50)
50    READ 2,N
 2    FØRMAT (I2)
      READ3, (X(I), I = 1,N)
      READ3, (Y(I) , I = 1,N)
 3    FØRMAT (8F10.5)
      READ4, (F(I), I = 1,N)
 4    FØRMAT (26F3.0,2X)
      S1 = 0.
      S2 = 0.
      S3 = 0.
      S4 = 0.
      S5 = 0.
      S6 = 0.
      S7 = 0.
      S8 = 0.
      S9 = 0.
      DØ 5 I = 1,N
      S1 = S1 + F(I)
      H1 = X(I) * F(I)
      H2 = X(I) * H1
      H3 = X(I) * H2
      H4 = X(I) * H3
      H5 = Y(I) * H1
      H6 = Y(I) * H2
      H7 = Y(I) * F(I)
      H8 = Y(I) * H7
      S2 = S2 + H1
      S3 = S3 + H2
      S4 = S4 + H3
      S5 = S5 + H4
      S6 = S6 + H5
      S7 = S7 + H6
      S8 = S8 + H7
      S9 = S9 + H8
      C10 = - S2/S1
      A1 = S2 * S3 - S1 * S4
      A2 = S1 * S3 - S2 * S2
      C21 = A1/A2
      A1 = S2 * S4 - S3 * S3
      A2 = S1 * S3 - S2 * S2
      C20 = A1/A2
      B0 = S8/S1
```

```
      A1=S6 - S2 * S8/S1
      A2=S3 - S2 * S2/S1
      B1=A1/A2
      A1=S7 + C21 * S6 + C20 * S8
      A2= S5 + C21 * S4 + C20 * S3
      B2=A1/A2
      A=B2
      B=B1 + C21 * B2
      C=B0 - S2 * B1/S1 + B2 * C20
      PUNCH6, C10, C21, C20, B0, B1, B2, A, B, C
6     FØRMAT(3X,4HC10=E12.5,4HC21=E12.5,4HC20=E12.5/3X,4HB0 =E12.5,4HB1
     X =E12.5,4HB2 =E12.5/3X,4HA =E12.5,4HB =E12.5,4HC =E12.5)
      REG2 = B2 * B2 (C20 * S3 + C21 * S4 + S5)
      REG1= B1 * B1 * (S3 - S2 * S2/S1)
      SST = S9 - S8 * S8/S1
      SSR = SST - REG1 - REG2
      K1 = 1
      K2 = 1
      K3 = S1 - 3.
      AK3 = K3
      K4 = S1 - 1.
      SSRM = SSR/AK3
      R1 = REG1/SSRM
      R2 = REG2/SSRM
      PUNCH8
8     FØRMAT(1X/28X,25HREGRESSION ANALYSIS TABLE)
      PUNCH9
9     FØRMAT(5X, 62H . . . . . . . . . . . . . . . . . . . . . . . . . .
     X . . . . . . . . . .)
      PUNCH10
10    FØRMAT(6X,19HSØURCE ØF VARIATIØN,8X,5HS.SQ.,8X,4HD.F.,8X,7HM.S.SQ.
     X )
      PUNCH9
      PUNCH11,REG2,K1,REG2
11    FØRMAT(6X,18HDUE TØ REGRESSIØN1,6X,E12.4,4X,I5,4X,E12.4)
      PUNCH12,REG1,K2,REG1
12    FØRMAT(6X,18HDUE TØ REGRESSIØN2,6X,E12.4,4X,I5,4X,E12.4)
      PUNCH13,SSR,K3,SSRM
13    FØRMAT(6X,8HRESIDUAL,16X,E12.4,4X,I5,4X,E12.4)
      PUNCH9
      PUNCH14,SST,K4
14    FØRMAT(6X,5HTØTAL,19X,E12.4,4X,I5)
      PUNCH9
      PUNCH15,R1,R2
15    FØRMAT(1X/8X,3HR1=,E12.4,2X,3HR2=,E12.4)
      GØ TØ 50
      END
```

Program Used