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Computer Applications To The Solution Of Inventory Models

II. Probabilistic Models

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Contents

Introduction:

This memo. is a continuation of the previous memo. dealing with the computer applications to the solution of inventory problems (deterministic models) Here, however the demand for the goods as commodities is probabilistic and is specified by a given distribution function. The models presented here treat different inventory situations which are of practical applicability. The methods for solving the models are given. Also the computer program written in FORTRAN coding language is presented for the models. The reader will find the details for using each program so it could be used directly without further investiations about the method of programming.

Elements of Probabilistic Inventory Models:

The following sysmbols will be used as indicated below

Let :

C = Purchase cost per item.

y = amount ordered per period.

r = revenue per item.

v = salvage value per item.

h = holding cost/item/period.

p = penalty cost/item/period.

$\phi(\xi)$ = demand density function/period. $\xi \geq 0$

In the following models our objective will be to determine the optimal y which maximizes profit. In the previous models we have treated the same problem which minimizes costs. The difference between the two cases depends on the viewpoint of the decision-maker.

<u>MODEL I.</u> (One-Period model)

In this model it is assumed that demand is continued for one period (seasonal) only. An example of this would occur with style or fashion items. The total profit function in this case will be equal to the total expected revenue less the cost of purchasing the ordered amount and the holding and penalty costs. Expressing this mathematically the total expected profit is given by :

$$\text{Expected profit} = \int_0^y \left(r\xi + v\,(y-\xi) - h\,(y-\xi) \right) \phi(\xi)\, d\xi + \int_y^\infty \left[ry - p\,(\xi - y) \right] \phi(\xi)\, d\xi - cy$$

= Expected revenue when demand level is less than stock

+ Expected revenue when demand level is higher than stock - purchase cost.

The formulation of the above expected profit is determined so follows:

1. demand is less than stock level(i.e, $\xi < y$), in which case a holding cost is accumulated. This holding cost is given by $\int_0^y h\,(y-\xi)\,\phi(\xi)\, d\xi$. Since we assume that unsold item can be returned at a reduced price (v), then expected revenue from salvage is given by $\int_0^y v(y-\xi)\,\phi\, d\xi$. To get the revenue from selling ξ items this is given by $\int_0^y r\xi\phi(\xi)\, d\xi$. This gives the elements of the first integral.

2. demand is higher than stock level (i.e. $\xi > y$) in which case a penalty cost is accrued. This is given by $\int_y^\infty p(\xi - y)\,\phi(\xi)\, d\xi$. While the revenue in this case will be given by r y. These give the terms for the second integral.

To solve for optimal order quantity y , the above profit function is differentiated with respect to y. To do this in an integral equation we use the following formula:

$$\frac{d}{dy}\left(\int_{u_0(y)}^{u_1(y)} f(x,y)\,dx\right) = \int_{u_0(y)}^{u_1(y)} \frac{f(x,y)}{y}\,dx - f(u_0,y)\frac{du_0}{dy} + f(u_1,y)\frac{du_1}{dy}$$

So, applying this to the above equation gives :

$$\frac{d\ \text{profit}}{dy} = \int_0^y (v-h)\,\phi(\xi)\,d\xi - 0 + ry\,\phi(y)$$

$$+ \int_y^{\infty}(r+p)\,\phi(\xi)\,d\xi - ry\,\phi(y) + 0$$

$$- C \quad = 0$$

This gives

$$(v-h)\int_0^y \phi(\xi)\,d\xi + (r+p)\int_y^{\infty}\phi(\xi)\,d\xi = C$$

But Since $\int_0^y \phi(\xi)\,d\xi = 1 - \int_y^{\infty}\phi(\xi)\,d\xi$

Substituting, this gives,

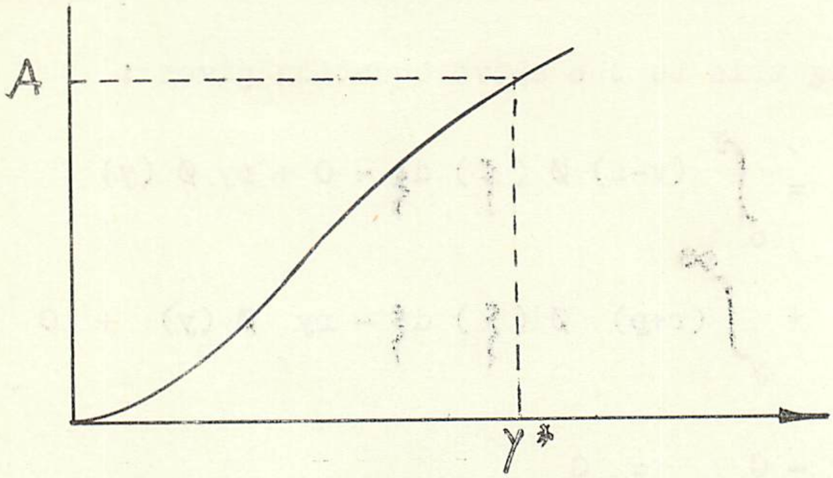
$$(v-h-r-p)\int_0^y \phi(\xi)\,d\xi + r+p = C$$

$$\text{or} \quad \int_0^y \phi(\xi)\,d\xi = \frac{r+p-c}{r+p+h-v} = A \quad \text{(say)}$$

The solution of this last equation is obtained by noticing that

$$P\left\{0 \leq \xi \leq y\right\} = \int_{0}^{y} \phi(\xi)\, d\xi = A$$

In other words, the optimal y^* can be determined so that the probability that demand $\xi \leq y$ is equal to the given value A. This is shown in Figure 1 below.



For the purpose of writing a computer program for solving this problem, we shall consider a special case where ξ takes discrete values. In this case it will be possible to feed in the different discrete values that can be assumed by ξ together with the corresponding probability of their occurence. This is shown by the following numerical example:

ξ	$P(\xi)$
1	.1
2	.2
3	.4
4	.3

now suppose

$$r = 10.$$
$$p = 5.$$
$$c = 8$$
$$h = 4$$
$$v = 6$$

hence $A = \frac{r+p-c}{r+p+h-v} = \frac{10+5-8}{10+5+4-6} = \frac{7}{13} = .5389$

From the above table, for the discrete demand case it is clear that the value of ξ satisfying optimal conditions is given by $\xi = y^*$,

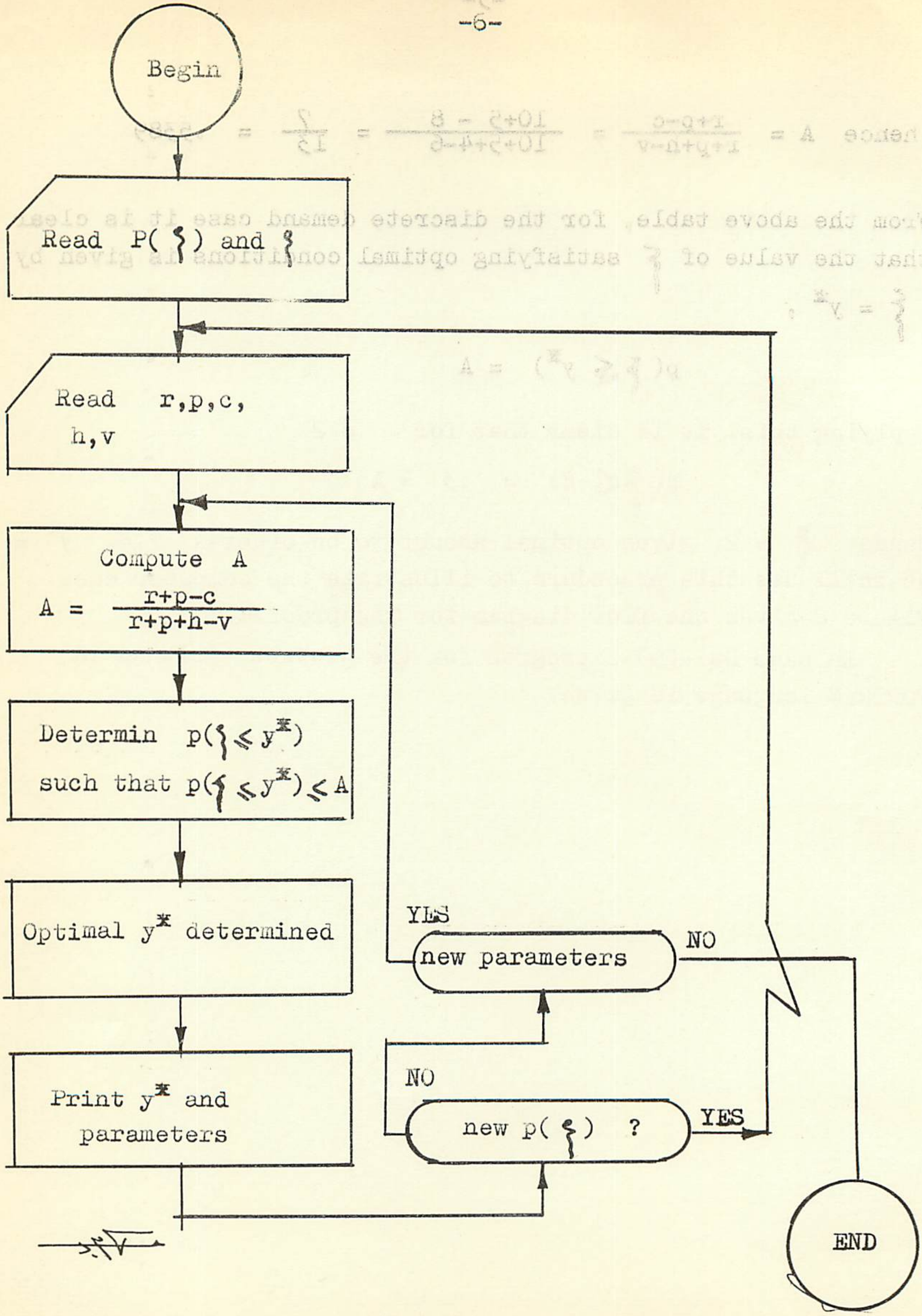
$$p(\xi \leqslant y^*) = A$$

Applying this, it is clear that for = 2

$$p(\xi \leqslant 2) = .3 = A$$

Hence $\xi = 2$ gives optimal amount to be ordered, i.e. $y^* = 2$. We shall use this procedure to illustrate the computer case. Figure 2 gives the flow diagram for the problem.

In page No. (9) a program for the problem, written in FORTRAN language is given.


Begin
Read P(ξ) and ξ
Read r,p,c, h,v
Compute A
A = (r+p-c)/(r+p+h-v)
Determin p(ξ ≤ y*) such that p(ξ ≤ y*) ≤ A
Optimal y* determined
Print y* and parameters
YES
new parameters
NO
NO
new p(ξ) ?
YES
END

Preparation of Input Data :

1. First card :

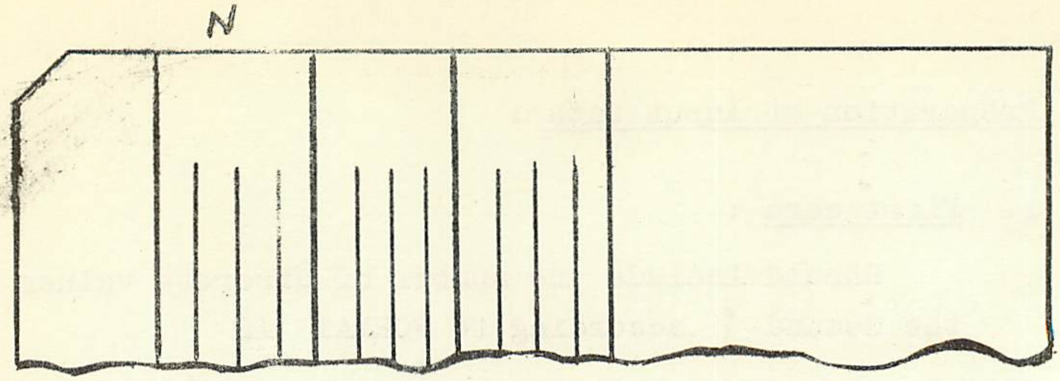
Should include the number of discrete values of the demand ξ according to FORMAT I2

2. Second N cards:

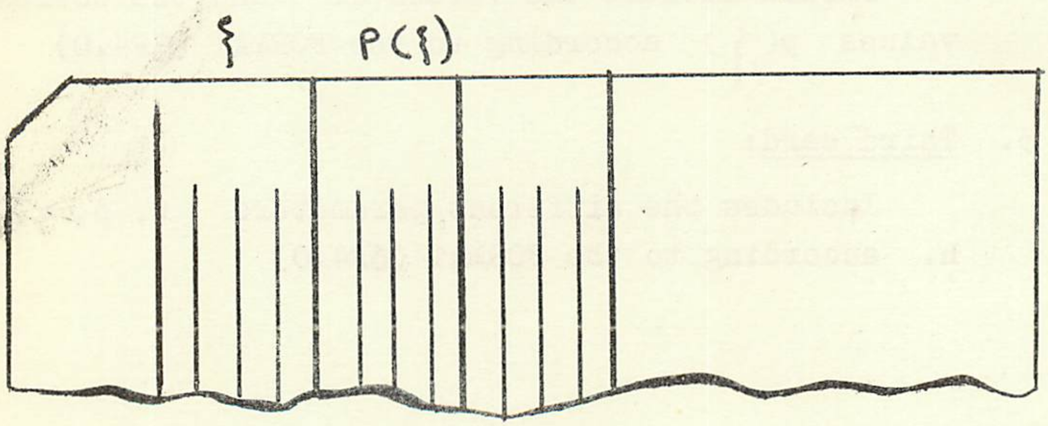
Should include the values of ξ and the corresponding values $p(\xi)$ according to the FORMAT (5F4.0)

3. Third card:

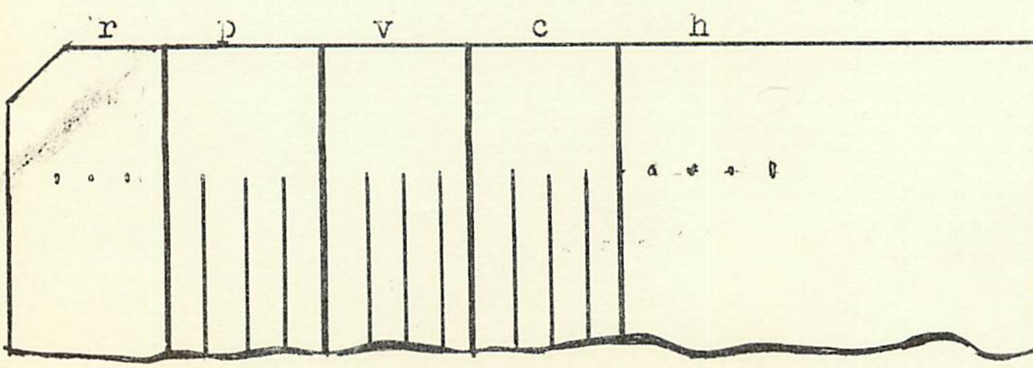
Includes the different parameters r, p, v, c, and h. according to the FORMAT (5F4.0)



First Card



Second Cards



Third Card

```
DIMENSIOND(20,20)

1 FORMAT(5F4.1)
2 FORMAT(/////31HPROBABILISTIC DEMAND - MODEL 1.)
3 FORMAT(/19HHOLDING COST/UNIT =,E11.4)
4 FORMAT(/19HPENALTY COST/UNIT =,E11.4)
5 FORMAT(20HSALVAGE VALUE/UNIT =,E11.4)
6 FORMAT(22HPURCHASING COST/UNIT =,E11.4)
7 FORMAT(14HREVENUE/UNIT =,E11.4)
8 FORMAT(//11HOPTIMAL Y =,I6)
9 FORMAT(I2)
17 FORMAT(2F4.1)
15 READ9,N
   DO16I=1,N
16 READ17,(D(I,J),J=1,2)
   READ1,R,P,V,C,H
   A=(R+P-C)/(R+P+H-V)
   SUM=0.
   DO11I=1,N
   SUM=SUM+D(I,2)
   IF(SUM-A)12,13,11
12 K=I+1
   CUM=SUM+D(K,2)
   IF(CUM-A)11,20,13
20 I=K
   GO TO 13
11 CONTINUE
13 NOPT=D(I,1)
```

```
PRINT2
PRINT3,H
PRINT4,P
PRINT5,V

PRINT6,C
PRINT7,R
PRINT8,NOPT
GO TO 15
END
```

ILLUSTRATIVE SOLVED EXAMPLE

PROBLEM NO. 1

PROBABILISTIC DEMAND - MODEL 1.

HOLDING COST/UNIT = 8.0000E-01

PENALTY COST/UNIT = 2.0000E-00
SALVAGE VALUE/UNIT = 2.8000E-00
PURCHASING COST/UNIT = 3.0000E-00
REVENUE/UNIT = 5.0000E-00

OPTIMAL Y = 4

PROBLEM NO. 2

PROBABILISTIC DEMAND - MODEL 1.

HOLDING COST/UNIT = 8.0000E-01

PENALTY COST/UNIT = 2.0000E-00
SALVAGE VALUE/UNIT = 2.8000E-00
PURCHASING COST/UNIT = 3.0000E-00
REVENUE/UNIT = 5.0000E-00

OPTIMAL Y = 6

MODEL II. (Infinite number of periods)

In this model we assume an infinite number of periods. We assume that delivery of ordered amount is immediate and that lost demand is backloged.

Let y = amount on hand after an order
x = " " " before an order
α = discount rate per period.

$$= \frac{1}{1+\text{ interest rate}}$$

Let f(x) be the optimal expected profit. This is given by

$$f(x) = \max_{y \geq x} \Big\{ - c(y-x) + \int_0^y (r\xi - h(y-\xi)\phi(\xi)d\xi + \int_y^\infty \left[r y + \alpha r(\xi - y) - p(\xi - y)\right]\phi(\xi)d\xi + \alpha \int_0^\infty f(y-\xi)\phi(\xi)\, d\xi$$

Solving for the value of y using the same procedure given in Model I above. This gives:

$$\int_0^{y^*} \phi(\xi)d\xi = \frac{p + (1-\alpha)\,(r-c)}{p + h + (1-\alpha)\, r}$$

The solution of this problem follows an exact procedure as in model I so we do not need to repeat it here.

CONCLUSION:

The problems presented here show an application to a real problem that is encountered in production and warehousing situation. The computer can further be used for solving more complex problems. However, we have presented some simple problems to get a feeling of the use of the computer.

References

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   Introduction & Operations Research New York. Wiley, 1959.