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A Developed Simplex Algorithm For Solving Transportation Problem

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Introduction.

As an economic interpretation of the linear programming model encompassing a wide variety of applications is the following. The modeled system includes several activities which shares limited resources. The objective of the model is to determines the level of each activity that optimizes the output of all activities without violating the limits stipulated on the resources.

The classical "transportation" and "assignment problems have the fortunate property that the linear programming solution or more precisely, every extrem point solution automatically assigns integer values to the variables. In this paper there is a representation of a transportation problem with initial and variable costs. The solution of such problem depends originaly on the distribution method which helps to get an initial feasible solution.

This initial solution is through a techniques developed from the simplex algorthim used and we get an optimal solution.

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1. Transportation Problem With intial cost.

1.1. The Nature of the Problem.

There is a special class of linear programming models that are of particular interest because of their structures as well as applications. The transportation model deals with the transportation of a commodity from m sources to n destinations. The supply available at source i is a_i units and the demand required at destination j is b_j units. X_{ij} will represents the level of activity (amount transported) from source i to destination j. The case to be studied heir will take incondedriation the fixed cost and also the interval fixed cost which is a new extension in dealing with the transportation problems. To do this let us assume the pollowing.

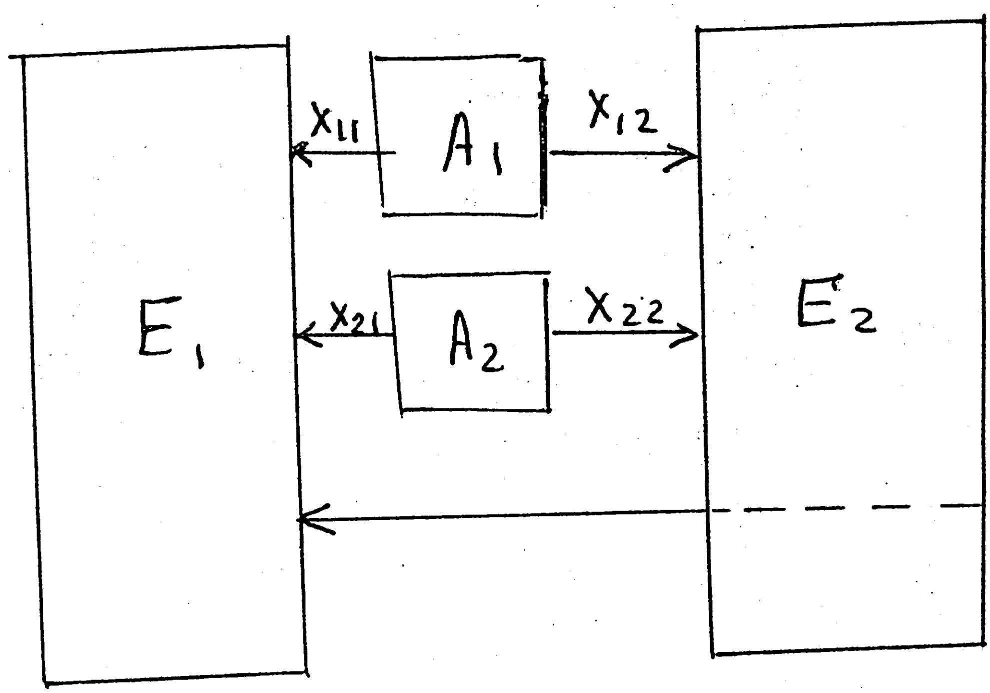
Given A_i sources (i=1,2,3) and E_j destinations (j=1,2) and it is asked about the minimum transport cost from A_i to E_j. Let T_1 and T_1^2 represents the arrangement of the transport standsill of the given places. Also it is assumed that we have two types of costs

a. Fixed costs K_i

b. Interval Fixed costs K_{ij}

These two costs are needed also for the transportation of a unit. The transportation problem is now considered as a mixed transportation problem, given the transports mittel(transit transport mittel) as t_i, which means that there is for the amount transported from A_i to E_j (X_{ij}) X_{11}, X_{12}, X_{21} and X_{22} transport mittel to be fullfild. This means that the transport-mittels allows only on E_j to be supplied, IF we want to supply the other E_j, then there must be new transport mittel.

This condition is also applied for A_3. IF for example we have a transport mittel for supplying from A_3, so the transport mittel is divided between E_1 and E_2 as seen in Fig(1). This condition is not applied for A_3, i.e. new transport mittel must be used.



Fig(1)

Beside the fixed costs and the interval fixed cost there exist also the transport cost for transporting a unit from A_i to E_j which are given by the following table

C_{ij}	E_1	E_2
A_1	100	120
A_2	100	100
A_3	160	110

Also the fixed cost for transporting from A_1, A_2 and A_3 will be

$$K_1 = 1500 \quad , \quad K_2 = 1680 \quad , \quad K_3 = 1200$$

The order of transports mittel for the standing A_1, A_2 and A_3 will be given by

$$T_1^1, T_1^2, \ldots\ldots \quad = 2u$$

$$T_2^1, T_2^2, \ldots\ldots \quad = 3u$$

$$T_3^1, T_3^2, \ldots\ldots \quad = 2u$$

For a seperate transports mittel for the given sources A_1 and A_3 the fixed cost will be of maximum value of 1000 and for the source A_2 will be of a value of 1320. This fixed cost of the transports mittel will be the total transport plan for the intervals

$$0,2,4 \quad x_{11}, x_{12}, x_{11}, x_{32} \quad 2,4,6$$

$$k_{11} = k_{12} = k_{31} = k_{32} = 1000$$

but for the source A_2 we assume that

$$k_{2j} = 1320 \qquad \text{for an interval of 3u}$$

Let now a_i represents the capacity of A_i and e_j be the needed sum from E_j then we have

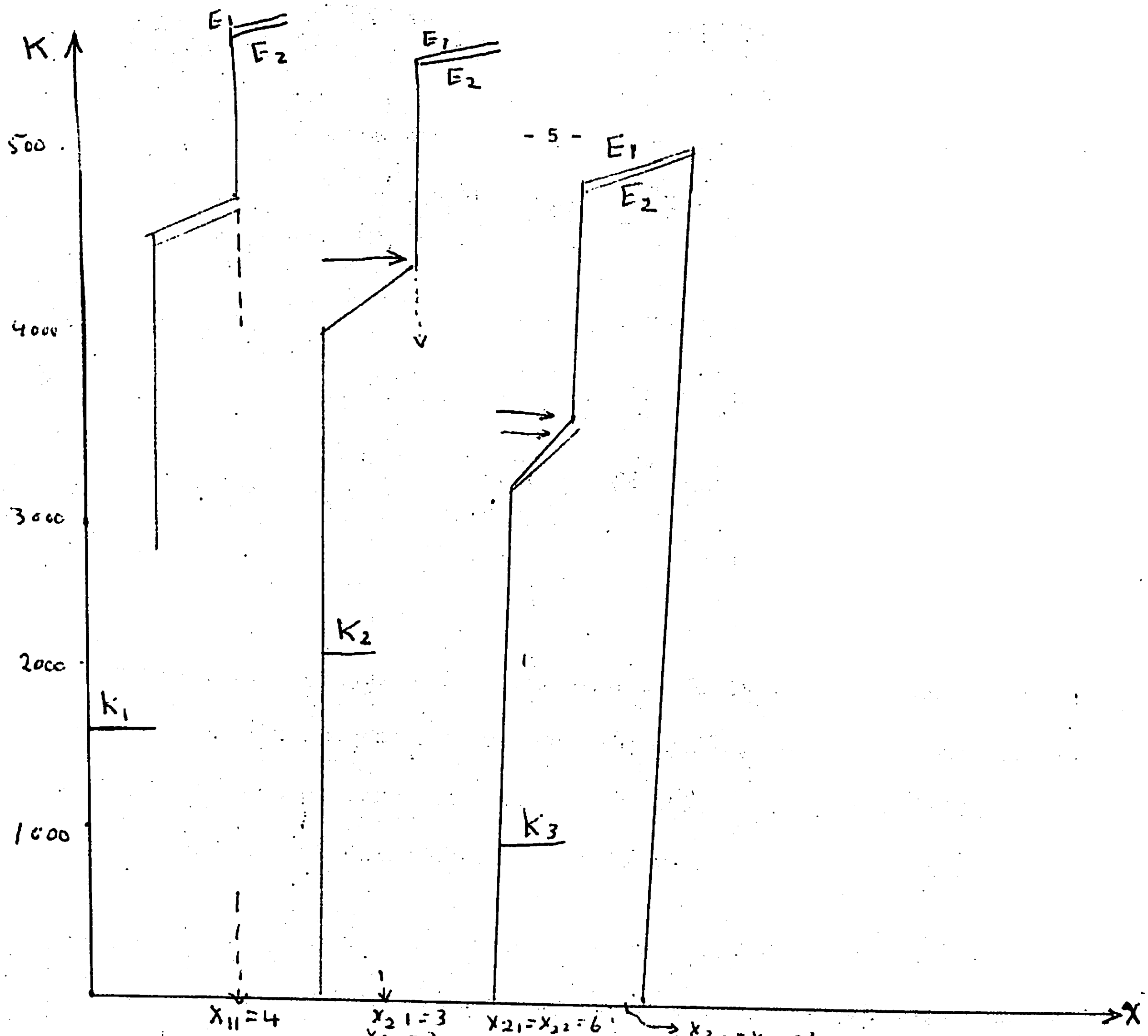
$$a_1 = 5 \qquad e_1 = 5$$

$$a_2 = 6 \qquad e_2 = 5$$

$$a_3 = 4$$

Since the total sum of a_i greater than that of e_j, then we assume that there is a thrid destination E_3 without transport costs ($C_{i3} = 0$).

The following fig (2) gives an illustration of the cost run for a seperate supply of one source.



Fig(2)

3. The Formulation of the Objective Function.

The transport cost function includes the total variable transport costs

$$K_v = \sum_i \sum_j c_{ij} \ x_{ij} \qquad (1)$$

The transport cost theory with the matrix C_{ij} as the costs of transportation.

For the total fixed costs for supplier resources we have

$$\sum K_i = K_1 v_1 + K_2 v_2 + K_3 v_3 \qquad (2)$$

with

$$x_{11} + x_{12} \leqslant a_1 v_1 \qquad (3a)$$

$$x_{21} + x_{22} \leqslant a_2 v_2 \qquad (3b)$$

$$x_{31} + x_{32} \leqslant a_3 v_3 \qquad (3c)$$

and

$$v_1, v_2, v_3 = \begin{Bmatrix} 0 \\ 1 \end{Bmatrix} \qquad (3d)$$

The fixed cost K_1 is real only, when there is a supply from a given source. Also we can see for example that when A_1 supplies either A_1 $(x_{11} \geqslant o)$ or/and E_2 $(x_{12} \geqslant 0)$, then v must be equal to one this is from (3d), which gives K_1 as the total cost and relation (2) is fullfiled.

Relation (3b) to (3c) represents the fixed costs K_2 and K_3.

For the interval fixed costs in the position A_1 we have

$$K^{1}_{jt} = k_{11} W^{1}_{1} + k_{12} W^{2}_{1} \qquad (4)$$

$$x_{11} \leqslant 2w^{1}_{1} \qquad (4a)$$

$$x_{12} \leqslant 2w^{2}_{1} \qquad (46)$$

$$w^{1}_{1}, w^{2}_{1} = 0 \qquad (4c)$$

$$w^{1}_{1}, w^{2}_{1} \geqslant 0 \qquad (4d)$$

The interval fixed cost k_{11} given by relation (4) is for example availble, when $0 \leqslant x_{11} \leqslant 2$, but if $2 \leqslant x_{11} \leqslant 4$, then the value of w^{1}_{1} must be at least equal to 2, this can be deduced from relations (4a) with (4c) and (4d).

Similarly for k_{12} we get a set of relations coresponding to A_1 and A_3 as follows.

$$K^{2}_{jt} + K^{3}_{jt} = k_{21} w^{1}_{2} + k_{22} w^{2}_{2} + k_{3} w_{3} \qquad (5)$$

with

$$x_{21} \leqslant 3w^{1}_{2} \qquad (5a)$$

$$x_{22} \leqslant 3w^{2}_{2} \qquad (5b)$$

$$x_{31} + x_{32} \leqslant 2w_{3} \qquad (5c)$$

$$w^{1}_{2}, w^{2}_{2}, w_{3} = 0 \qquad (5d)$$

$$w^{1}_{2}, w^{2}_{2}, w_{3} \geqslant 0 \qquad (5e)$$

Relations (5) and (5c) shows that the source A_3 supply the destinations E_1 and E_2 only in one direction, which means that only one transport mittel is used. In this case we see that the interval transport costs are divided between x_{31} and x_{32}.

Also relations (4) to (5c) shows that there is no interval fixed costs for the supply of destination E_3.

The final objective function which is the total cost function is formulated as follows

$$K = \sum_i \sum_j a_{ij} x_{uj} + \sum_i K_i v_i + \sum_{j=1}^{2} k_{ij} w_1^j + \sum_{j=1}^{2} k_{2j} w_2^j + k_3 w_3 \qquad (6)$$

In the above cost function (6) the set of constraints (3a) to (3e); (4a) to (4d) and (5a) to (5e) are to be taken into consideration.

1.3 The Constraints

The objective function (6) is formulated without any condition on the variables x_{ij} (the non-negativity condition on the variables). The capacity conditions are included in the constraints (3a), (3b) and (3c).

Beside the non-negativity of the variables, we most also represents the supply demand conditions between A_i and E_j which gives

$$\sum_i x_{ij} \geq e_j \tag{7}$$

If we introduce the variable V_i, which means that if the supply of A_i exist then the value of V_i is equal to 1, and the value of x_{ij} is not greater than a_i.

Now we are in a position to get the mathematical formulation of the problem as follow.

Given an objective function

$$\begin{aligned} K = & 100x_{11} + 120x_{12} + 100x_{21} + 100x_{22} + 160x_{31} \\ & + 110x_{32} + 1500V_1 + 1660V_2 + 1200V_3 \\ & + 1000w_1^1 + 1000w_1^2 + 1320w_2^1 + 1320w_2^2 \\ & + 1000w_3 \ldots\ldots\ldots \end{aligned} \tag{8}$$

is to be maximized under the constraints.

$$x_{11} + x_{12} - 5v_1 \leq 0 \tag{9}$$

$$x_{21} + x_{22} - 6v_2 \leq 0 \tag{10}$$

$$x_{31} + x_{32} - 4v_3 \leq 0 \tag{11}$$

$$x_{11} - 2w_1^1 \leq 0 \qquad (12)$$

$$x_{12} - 2w_1^2 \leq 0 \qquad (13)$$

$$x_{21} - 3w_2^1 \leq 0 \qquad (14)$$

$$x_{22} - 3w_2^2 \leq 0 \qquad (15)$$

$$x_{31} + x_{32} - 2w_2 \leq 0 \qquad (16)$$

$$x_{11} + x_{21} + x_{31} \geq 5 \qquad (17)$$

$$x_{12} + x_{22} + x_{32} \geq 5 \qquad (18)$$

$$v_1, v_2, v_3 \leq 1 \qquad (19)$$

$$v_1, v_2, v_3, w_1^1, w_1^2, w_3 = 0 \qquad (20)$$

$$\left.\begin{array}{l} x_{ij} \geq 0 \\ v_i \geq 0 \quad \begin{pmatrix} i=1,2,3 \\ j=1,2 \end{pmatrix} \\ w_i^j \geq 0 \end{array}\right. \qquad (21)$$

If we use the simplex method to get a solution for the above problem, this means that we are dealing with 14 basic variables and 13 slack variables due to the constraints from (9) to (19). But in case of using the M-Method there must be two additional salack variables. For the two constraints (17) and (18) i.e. our problem consists of 29 variables.

1.4. The Solution

To solve this transportation problem we begin using the simplex-method (to get an optimal table). But since the constraints (17) and (18) dose not includes basic feasible variables then the simplex method must be devolped in order to overcome this situation. Which mean that the simplex method in its normal form dose not gives a solution for this problem. Also if we use the M-technique or the two-phase method, we find that an intial feasible solution is obtained after a large number of steps.

To avoid this we begin by using the distribution-method to get an optimal table . This optimal table is used to get an intial feasible solution by applying the developed simplex method.

This intial feasible solution is obtained after small number of calculation.

The non-integer optimal solution obtained by the simplex method gives also non-negative solution for v_i and w_i. This means that the fixed costs and the interval costs must be proportional. Then in order to use the simplex method to get a result, we must at first get the proportionality of the fixed cost and interval cost. This proportionality can be as follows

$$\frac{1500}{5} = 300 \; ; \quad \frac{1680}{6} = 280 \; , \quad \frac{1200}{4} = 300$$

which gives also the fixed costs per transport mittel.

For sources A_1 and A_3 we have

for A_1 $\frac{1000}{2} = 500$ and for A_2 $\frac{1300}{3} = 440$

If we add the proportional fixed costs and the interval costs to the transport costs c_{ij}, we get the following transport cost matrix (c_{ij}).

c'_{ij}	E_1	E_2	E_3
A_1	900	920	0
A_2	820	820	0
A_3	960	910	0

As it can be seen that E_3 is to be considered with zero costs. In order to get an intial solution for the problem we begin by using the north-west rule, then we obtain the following.

1	E_1	E_2	E_3	
A_1	5	+20	-80	5
A_2	0	5	1	6
A_3	+140	+90	4	4
	5	5	5	15

As it can be seen from the above table, there is some numbers with signs.

These numbers gives the removal advantage and disadvantage. Now we move to the second step which enables to get an optimal table as follows.

2	E_1	E_2	E_3	
A_1	4	+20	1	5
A_2	1	5	+80	6
A_3	+40	+10	4	4
	5	5	5	15

The solution obtined is an optimal solution, and gives the fixed cost proportionality as shown in Fig(2). The optimal table (2) shows that there is a removal disadvantage. This solution gives values for x_{ij} as follows.

$x_{11} = 4$, $x_{12} = 0$, $x_{13} = 1$

$x_{21} = 1$, $x_{22} = 5$, $x_{23} = 0$

$x_{31} = 0$, $x_{32} = 0$, $x_{33} = 4$

To continue our solution we begin by using the simplex table, In this case relations (9) to (19) are used taking intoconsideration the non-negativity of the variables y_1 and y_{13}
Now solving for the non-basic variables we get.

$$y_1 = -x_{11} - x_{12} + 5v_1 \quad (9)$$

$$y_2 = -x_{21} - x_{22} + bv_2 \quad (10)$$

$$y_3 = -x_{31} - x_{32} + 4v_3 \quad (11)$$

$$y_4 = -x_{11} + 2w_1^1 \quad (12)$$

$$y_5 = -x_{12} + 2w_1^2 \quad (13)$$

$$y_6 = -x_{21} + 3w_2^1 \quad (14)$$

$$y_7 = -x_{22} + 3w_2^2 \quad (15)$$

$$y_8 = -x_{31} - x_{32} + 2w_3 \quad (16)$$

$$y_9 = -5 + x_{11} + x_{21} + x_{31} \quad (17)$$

$$y_{10} = -5 + x_{12} + x_{22} + x_{32} \quad (18)$$

$$y_{11} = 1-v_1 \quad (19a)$$

$$y_{12} = 1-v_2 \quad (19b)$$

$$y_{13} = 1-v_3 \quad (19c)$$

The above system of equations is conected with that deduced from (17) of the, intial simplex table in studying the maximized negative cost function

$$-K = -100x_{11} - 120x_{12} - 100x_{21} - 100x_{22}$$

$$-160x_{31} - 110x_{32} - 1500x_1 - 1680v_2$$

$$-1000w_1^1 - 1000w_1^2 - 1320w_2^1 - 1320w_2^2 - 1000w_3$$

In order to define the basis of the optimal solution, we must get a solution of the system (9) — (19) with the above objective function. (all the variables of the solution must be greater than zero).

Using the distribution method the solution gives the values of ($x_{11}x_{21}$, x_{22}, x_{33}).

If the variables x_{11}, x_{21}, x_{22} are greater than zero, then the variables v_1 and v_2 must be greater than zero. The same will be for the variables w_1 , w_2 and w_2^2 (i.e. w_1^1 , w_2^1 and w_2^2 must be greater than zero, from equations (12), (14) and (15).

In the optimum case are all artifial variables (y_1 to y_8) equal to the value zero, this is since the values of v and w are of large values.
If some values of v or w takes large values (not all/then the coresspond-ing artificial variables will be greater than zero.

The artificial variable y_{12} is equal to zero. Since A_2 which represents the total capacity are

$$(x_{21} + x_{22} = 6, \qquad x_{23} = 0)$$

The variable v_2 is then equal to one and y_{12} will be equal to zero (from (19 b).

Also v_1 and v_3 are smaller than one this is because that the amounts supplied by sources A_1 and A_3 is less than its capacity. This means that y_{11} and y_{13} greater than zero, this is obtained from relation (19a) and (19b).

The basic variables are then all o, these basic variables are

$$x_{11} \quad , \quad v_1 \quad , \quad w_1^1 \quad , \quad y_{11}$$

$$x_{21} \quad , \quad v_3 \quad , \quad w_2^1 \quad , \quad y_{13}$$

$$x_{22} \qquad\qquad\qquad w_2^2$$

those 10 variables are now defined since we have now 13 equations which represents the bounds of the system, then 13 variables must be as the basic variable, for this reason we introduce the variables v_2 , w_1^2 , x_{23} as basic variables.

The variables x_{13}, x_{23} and x_{33} which are considered as artificial variables in the distribution method represents the variables y_1 , y_2 and y_3 when v_1 , v_2 and v_3 are equal to zero.

If we solve the system of equations (9) to (19c) with respect to 13 variables, this means that we are begining with the artificial variables.

For y_{11} we get the form

$$y_{11} = 1-v_1 \tag{19a}$$

Since v_1 is always basic then it will be defined from (9) with

$$v_1 = \frac{1}{5} x_{11} + \frac{1}{5} x_{12} + \frac{1}{5} y_1 \qquad (9)$$

Also the variable x_{11} is can be deduced from relation (17) that

$$x_{11} = 5 - x_{21} - x_{31} + y_9 \qquad (17)$$

Since the above relation includes the basic variable x_{21}, then its expression will be obtained from (10) as

$$x_{21} = -x_{22} + 6\, v_2 - y_2 \qquad (10)$$

and the same is to be done for x_{22} from (19) and for v_2 from (19b) then we have

$$\underline{x_{22} = 5 - x_{12} - x_{32} + y_{10}} \qquad (18')$$

$$\underline{v_2 = 1 - y_{12}} \qquad (19'b)$$

Both expresions for x_{22} and v_2 are expressed by non-basic variables Relations (18') and (19') are taken in their form in the simplex table The same is done for the other variable, for x_{21} we use (18') and (19') with (10) we get.

$$\underline{x_{21} = 1 + x_{12} + x_{32} - y_2 - y_{10} - by_{12}} \qquad (10')$$

From relation (10') and (17) we get for x_{11} the following.

$$x_{11} = 4 - x_{12} - x_{31} - x_{32} + y_2 + y_9 + y_{10} + 6y_{12} \qquad (17)$$

putting this in relation (9), then we get for v_1

$$v_1 = \frac{4}{5} - \frac{1}{5} x_{31} - \frac{1}{5} x_{32} + \frac{1}{5} y_1 + \frac{1}{5} y_2 + \frac{1}{5} y_9 + \frac{1}{5} y_{10} + \frac{6}{5} y_{12} \qquad (9')$$

From (19a) and (9') we get for y_{11}

$$\underline{y_{11} = \frac{1}{5} + \frac{1}{5} x_{31} + \frac{1}{5} x_{32} - \frac{1}{5} y_2 - \frac{1}{5} y_2}$$

$$\underline{- \frac{1}{5} y_9 - \frac{1}{5} y_{10} - \frac{6}{5} = y_{12}} \qquad (19'a)$$

we are left now with 6 basic variables as non-basic variables.

The next step is to select y_{13} as a basic variable then from (19c) we have

$$y_{13} = 1 - \underline{v}_3 \qquad (19c)$$

for the variable v_3 we get from

$$v_3 = \frac{1}{4} x_{31} + \frac{1}{4} x_{32} + \frac{1}{4} y_3 \qquad (11')$$

Putting (11') in (19c) then we get for y_{13}.

$$y_{13} = 1 - \frac{1}{4} x_{31} - \frac{1}{4} x_{32} - \frac{1}{4} y_3 \qquad (19'c)$$

For the variables, w_1^1 , w_1^2 , w_2^2 and w_3 as basic variable we have the following.

$$w_1^1 = 2- \frac{1}{2} x_{12} \frac{1}{2} x_{31} - \frac{1}{2} x_{32} + \frac{1}{2} y_2 + \frac{1}{2} y_4$$

$$+ \frac{1}{9} y_9 + \frac{1}{2} y_{10} + 3y_{12} \qquad (12')$$

$$w_1^2 = \frac{1}{2} x_{12} + \frac{1}{2} y_5 \qquad (13')$$

$$w_2^1 = \frac{1}{3} + \frac{1}{3} x_{12} + \frac{1}{3} x_{32} - \frac{1}{3} y_2 - \frac{1}{3} y_6$$

$$- \frac{1}{3} y_{10} - 2y_{12} \qquad (14')$$

$$w_2^2 = \frac{5}{3} - \frac{1}{3} x_{12} - \frac{1}{3} x_{32} + \frac{1}{3} y_7 + \frac{1}{3} y_{10} \qquad (15')$$

$$w_3 = \frac{1}{2} x_{31} + \frac{1}{2} x_{32} + \frac{1}{2} y_8 \qquad (16')$$

The objective function will be after introducing the basic variables obtained above as follows

$$K = -8520 -20x_{12} - 60x_{31} - 10x_{32}$$

$$-300 y_1 - 360 y_2 - 500 y_4 - 500 y_5 -440y_6 -440y_7 -500y_8 -900y_{10}$$

$$-500 y_{10} - 480 y_{12}.$$

After this formulation of the objective function and the non-basic variables we can apply the simplex form in a developing way to get the non-integer optimal solution, the solution can be seen from the tables from 0 to 8 .

The optimal solution is then obtained by using the distribution method and the developed simplex algorthim. This solution is as follows

$v_2 = 1$ $\quad$ $w_2^1 = 1$

$x_{21} = 3$ $\quad$ $v_3 = 1$ $\quad$ $w_2^2 = 1$

$x_{22} = 3$ $\quad$ $3_3 = 2$

$x_{31} = 2$

$x_{32} = 2$

Fig (2) shows the solution of the problem.

●		x_{12}	x_{31}	x_{32}	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	y_9	y_{10}	y_{12}
$-K=$	-8520	20	60	10	300	360	300	500	500	440	440	500	900	900	480
$X_{11}=$	4	1	1	1		-1							-1	-1	-6
$X_{21}=$	1	-1		-1		1								1	6
$X_{22}=$	5	1		1										-1	
$V_1=$	$\frac{4}{5}$		$\frac{1}{5}$	$\frac{1}{5}$	$-\frac{1}{5}$	$-\frac{1}{5}$							$-\frac{1}{5}$	$-\frac{1}{5}$	$-\frac{6}{5}$
$V_2=$	1														1
$V_3=$	0		$-\frac{1}{4}$	$-\frac{1}{4}$			$-\frac{1}{4}$								
$W_1^1=$	2	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$		$-\frac{1}{2}$		$-\frac{1}{2}$					$-\frac{1}{2}$	$-\frac{1}{2}$	-3
$W_1^2=$	0	$-\frac{1}{2}$							$-\frac{1}{2}$						
$W_2^1=$	$\frac{1}{3}$	$-\frac{1}{3}$		$-\frac{1}{3}$		$\frac{1}{3}$				$-\frac{1}{3}$				$\frac{1}{3}$	2
$W_2^2=$	$\frac{5}{3}$	$\frac{1}{3}$		$\frac{1}{3}$							$-\frac{1}{3}$			$-\frac{1}{3}$	
$W_3=$	0		$-\frac{1}{2}$	$-\frac{1}{2}$								$-\frac{1}{2}$			
$y_{11}=$	$\frac{1}{5}$		$-\frac{1}{5}$	$-\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$							$\frac{1}{5}$	$\frac{1}{5}$	$\frac{6}{5}$
$y_{13}=$	1		$\frac{1}{4}$	$\frac{1}{4}$			$\frac{1}{4}$								
$S_1=$	$-\frac{2}{3}$	$-\frac{1}{3}$		$(-\frac{1}{3})$							$-\frac{2}{3}$			$-\frac{2}{3}$	

	E_1	E_2	E_3	a_i
A_1	4			5
A_2	1	5		6
A_3			4	4
e_j	5	5	5	15

1		x_{12}	x_{31}	s_1	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	y_9	y_{10}	y_{12}
$-k=$	-8540	10	60	30	300	360	300	500	500	440	420	500	900	880	480
$x_{11}=$	2		1	3							-2		-1	-3	-6
$x_{21}=$	3			-3		1					2			3	6
$x_{22}=$	3			3							-2			-3	
$v_1=$	$\frac{2}{5}$	$-\frac{1}{5}$	$-\frac{1}{5}$	$\frac{3}{5}$	$-\frac{1}{5}$	$-\frac{1}{5}$					$-\frac{2}{5}$		$-\frac{1}{5}$	$-\frac{3}{5}$	$-\frac{6}{5}$
$v_2=$	1														1
$v_3=$	$\frac{1}{2}$	$\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{3}{4}$			$-\frac{1}{4}$				$\frac{2}{4}$			$\frac{2}{4}$	
$w_1^1=$	1		$\frac{1}{2}$	$\frac{3}{2}$		$-\frac{1}{2}$		$-\frac{1}{2}$			-1		$-\frac{1}{2}$	$-\frac{3}{2}$	-3
$w_1^2=$	0	$-\frac{1}{2}$							$-\frac{1}{2}$						
$w_2^1=$	1			-1		$\frac{1}{3}$				$-\frac{1}{3}$	$\frac{2}{3}$			1	2
$w_2^2=$	1			1							-1			-1	
$w_3=$	1	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{3}{2}$							1	$-\frac{1}{2}$		1	
$y_{11}=$	$\frac{3}{5}$	$\frac{1}{5}$	$-\frac{1}{5}$	$-\frac{3}{5}$	$\frac{1}{5}$	$\frac{1}{5}$					$\frac{2}{5}$		$\frac{1}{5}$	$\frac{3}{5}$	$\frac{6}{5}$
$y_{12}=$	$\frac{1}{2}$	$-\frac{1}{4}$	$\frac{1}{4}$	$\frac{3}{4}$			$\frac{1}{4}$				$-\frac{1}{2}$			$-\frac{1}{2}$	
$y_{32}=$	2	1		-3							2			2	
$s_2=$	$-\frac{3}{5}$	$(-\frac{1}{5})$	$-\frac{4}{5}$	$-\frac{3}{5}$	$-\frac{1}{5}$	$-\frac{1}{5}$					$-\frac{2}{5}$		$-\frac{1}{5}$	$-\frac{3}{5}$	$-\frac{6}{5}$

	E_1	E_2	E_3	a_i
A_1	2		3	5
A_2	3	3		6
A_3		2	2	4
e_j	5	5	5	15

z		S_2	X_{31}	S_1	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	y_9	y_{10}	y_{12}
$-k=$	-8570	50	20	10	290	350	300	500	500	440	400	500	890	850	470
$X_{11}=$	2	0	1	3	0	0					-2		-1	-3	-6
$X_{21}=$	3			-3		1					2			3	6
$X_{22}=$	3			3							-2			-3	
$V_1=$	1	-1	1	1											-1
$V_2=$	1														1
$V_3=$	$-\frac{1}{4}$	$\frac{5}{4}$	$-\frac{5}{4}$	$-\frac{5}{4}$	$-\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{4}$						$-\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{4}$
$W_1^1=$	1		$\frac{1}{2}$	$\frac{3}{2}$		$-\frac{1}{2}$		$-\frac{1}{2}$			-1		$-\frac{1}{2}$	$-\frac{3}{2}$	-3
$W_1^2=$	$\frac{3}{2}$	$-\frac{5}{2}$	2	1	$\frac{1}{2}$	$\frac{1}{2}$			$-\frac{1}{2}$		1		$\frac{1}{2}$	$\frac{3}{2}$	$\frac{1}{2}$
$W_2^1=$	1			-1		$\frac{1}{3}$				$-\frac{1}{3}$	$\frac{2}{3}$			1	2
$W_2^2=$	1			1							-1			-1	
$W_3=$	$-\frac{1}{2}$	$\frac{5}{2}$	$-\frac{5}{2}$	$-\frac{5}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$						$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$
$y_{11}=$		1	-1	-1											1
$y_{13}=$	$\frac{5}{4}$	$-\frac{5}{4}$	$\frac{5}{4}$	$\frac{5}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$						$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$
$X_{32}=$	-1	5	-4	-5	-1	-1							-1	-1	-1
$X_{12}=$	3	-5	4	2	1	1					2		1	3	1

	E_1	E_2	E_3	a_i
A_1	2	3		5
A_2	3	3		6
A_3		-1	5	4
e_j	5	5	5	

3		s_2	x_{31}	v_3	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	y_9	y_{10}	y_{12}
$-K=$	-8572	60	10	8	288	348	298	500	500	440	400	500	888	848	468
$x_{11}=$	$\frac{7}{5}$	3	-2	$\frac{12}{5}$	$-\frac{3}{5}$	$-\frac{3}{5}$	$-\frac{3}{5}$				-2		$-\frac{8}{5}$	$-\frac{18}{5}$	$\frac{33}{5}$
$x_{21}=$	$\frac{18}{5}$	-3	3	$-\frac{12}{5}$	$\frac{3}{5}$	$\frac{8}{5}$	$\frac{3}{5}$				2		$\frac{3}{5}$	$\frac{18}{5}$	$\frac{33}{5}$
$x_{22}=$	$\frac{12}{5}$	3	-3	$\frac{12}{5}$	$-\frac{3}{5}$	$-\frac{3}{5}$	$-\frac{3}{5}$				-2		$-\frac{3}{5}$	$-\frac{18}{5}$	$-\frac{3}{5}$
$v_1=$	$\frac{4}{5}$			$\frac{4}{5}$	$-\frac{1}{5}$	$-\frac{1}{5}$	$-\frac{1}{5}$						$-\frac{1}{5}$	$-\frac{1}{5}$	$\frac{6}{5}$
$v_2=$	1														1
$w_1^1=$	$\frac{7}{10}$	$\frac{3}{2}$	-1	$\frac{6}{5}$	$-\frac{3}{10}$	$-\frac{8}{10}$	$-\frac{3}{10}$	$-\frac{1}{2}$			-1		$-\frac{8}{10}$	$-\frac{18}{10}$	$\frac{33}{10}$
$w_1^2=$	$\frac{13}{10}$	$-\frac{3}{2}$	1	$\frac{4}{5}$	$\frac{3}{10}$	$\frac{3}{10}$	$-\frac{1}{5}$		$-\frac{1}{2}$		1		$\frac{3}{10}$	$\frac{13}{10}$	$\frac{3}{10}$
$w_2^1=$	$\frac{6}{5}$	-1	1	$\frac{4}{5}$	$\frac{1}{5}$	$\frac{18}{15}$	$\frac{1}{5}$			$-\frac{1}{3}$	$\frac{2}{3}$		$\frac{1}{5}$	$\frac{6}{5}$	$\frac{1}{5}$
$w_2^2=$	$\frac{4}{5}$	1	-1	$\frac{4}{5}$	$-\frac{1}{5}$	$-\frac{1}{5}$	$-\frac{1}{5}$				-1		$-\frac{1}{5}$	$-\frac{6}{5}$	$-\frac{6}{5}$
$w_3=$	0			-2			$\frac{1}{2}$					$-\frac{1}{2}$			
$y_{11}=$	$\frac{1}{5}$			$-\frac{4}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$						$\frac{1}{5}$	$\frac{1}{5}$	$\frac{6}{5}$
y_{13}	1			1											
$y_{32}=$	0		1	-4			1								
$x_{12}=$	$\frac{13}{5}$	-3	2	$\frac{8}{5}$	$\frac{3}{5}$	$\frac{3}{5}$	$-\frac{2}{5}$						$\frac{3}{5}$	$\frac{13}{5}$	$\frac{3}{5}$
$s_3=$	$\frac{4}{5}$			$(-\frac{4}{5})$	$-\frac{4}{5}$	$-\frac{4}{5}$	$-\frac{4}{5}$	$-\frac{4}{5}$					$-\frac{4}{5}$	$-\frac{4}{5}$	$-\frac{4}{5}$

	E_1	E_2	E_3	a_i
A_1	$\frac{7}{5}$	$\frac{13}{5}$	1	5
A_2	$\frac{18}{5}$	$\frac{12}{5}$		6
A_3			4	4
e_j	5	5	5	15

4		s_2	x_{31}	s_3	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	y_9	y_{10}	y_{12}
$-k=$	-8580	60	10	10	280	340	290	500	500	440	400	500	880	840	460
$x_{11}=$	-1	3	(-2)	3	-3	-3	-3				-2		-4	-6	-9
$x_{21}=$	6	-3	3	-3	3	4	3				2		3	6	9
$x_{2\bar{2}}=$	0	3	-3	3	-3	-3	-3				-2		-3	-6	-3
$v_1=$	0			1	-1	-1	-1						-1	-1	-2
$v_2=$	1														1
$w_1^1=$	$-\frac{1}{2}$	$\frac{3}{2}$	-1	$\frac{3}{2}$	$-\frac{3}{2}$	-2	$-\frac{3}{2}$	$-\frac{1}{2}$			-1		-2	-3	$-\frac{9}{2}$
$w_1^2=$	$\frac{1}{2}$	$-\frac{3}{2}$	1	1	$-\frac{1}{2}$	$-\frac{1}{2}$	-1		$-\frac{1}{2}$		1		$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$
$w_2^1=$	2	-1	1	-1	1	2	1			$-\frac{1}{3}$	$\frac{2}{3}$		1	2	1
$w_2^2=$	0	1	-1	1	-1	-1	-1				-1		-1	-2	-1
$w_3=$	2			$-\frac{10}{4}$	2	2	$\frac{5}{2}$					$-\frac{1}{2}$	2	2	2
$y_{11}=$	1			-1	1	1	1						1	1	2
$y_{13}=$	0			$\frac{5}{4}$	-1	-1	-1						-1	-1	-1
$x_{3\bar{1}}=$	4		1	-5	4	4	5						4	4	4
$x_{1\bar{1}}=$	1	-3	2	2	-1	-1	-2						-1	1	-1
$v_{\bar{3}}=$	1			$\frac{5}{4}$	1	1	1						1	1	1

	E_1	E_2	E_3	a_i
A_1	-1	1	5	5
A_2	6			6
A_3		4		4
	5	5	5	15

5		s_2	x_{11}	s_3	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	y_9	y_{10}	y_{12}
$-k=$	-8585	75	5	25	265	325	275	500	500	440	390	500	860	810	415
$x_{31}=$	$\frac{1}{2}$	$-\frac{3}{2}$	$-\frac{1}{2}$	$-\frac{3}{2}$	$\frac{3}{2}$	$\frac{3}{2}$	$\frac{3}{2}$				1		2	3	$\frac{9}{2}$
$x_{21}=$	$\frac{9}{2}$	$\frac{3}{2}$	$\frac{3}{2}$	$\frac{3}{2}$	$-\frac{3}{2}$	$-\frac{1}{2}$	$-\frac{3}{2}$				-1		-3	-3	$-\frac{9}{2}$
$x_{22}=$	$\frac{3}{2}$	$-\frac{3}{2}$	$-\frac{3}{2}$	$-\frac{3}{2}$	$\frac{3}{2}$	$\frac{3}{2}$	$\frac{3}{2}$				1		3	3	$\frac{21}{2}$
$v_1=$	0			1	-1	-1	-1						-1	-1	-2
$v_2=$	1														1
$w_1^1=$	0		$-\frac{1}{2}$			$-\frac{1}{2}$		$-\frac{1}{2}$							
$w_1^2=$	0		$\frac{1}{2}$	$\frac{5}{2}$	-2	-2	$-\frac{5}{2}$		$-\frac{1}{2}$				$-\frac{5}{2}$	$-\frac{5}{2}$	-5
$w_2^1=$	$\frac{3}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$			$-\frac{1}{3}$	$-\frac{1}{3}$		-1	-1	$-\frac{3}{2}$
$w_2^2=$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$						1	1	$\frac{7}{2}$
$w_3=$	2			$-\frac{5}{2}$	2	2	$\frac{5}{2}$					$-\frac{1}{2}$	2	2	2
$y_{11}=$	1			-1	1	1	1						1	1	2
$y_{13}=$	0			$\frac{5}{4}$	-1	-1	-1						-1	-1	-1
$x_{32}=$	$\frac{7}{2}$	$\frac{3}{2}$	$\frac{1}{2}$	$-\frac{7}{2}$	$\frac{5}{2}$	$\frac{5}{2}$	$\frac{7}{2}$				-1		2	1	$-\frac{1}{2}$
$x_{12}=$	0		1	5	-4	-4	-5				-2		-5	-5	-10
$v_3=$	1			$-\frac{5}{4}$	1	1	1						1	1	1
$s_4=$	$-\frac{1}{2}$	$-\frac{1}{2}$	$(-\frac{1}{2})$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$								$-\frac{1}{2}$

	E_1	E_2	E_3	a_i
A_1			5	5
A_2	$\frac{9}{2}$	$\frac{3}{2}$		6
A_3	$\frac{1}{2}$	$\frac{7}{2}$		4
e_j	5	5	5	15

6		S_2	S_4	S_3	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	y_9	y_{10}	y_{12}
$-K=$	-850	70	10	20	260	320	270	500	500	440	390	500	860	810	410
$X_{31}=$	1	-1	-1	-1	2	2	2				1		2	3	5
$X_{21}=$	3		3		-3	-2	-3				-1		-3	-3	-6
$X_{22}=$	3		-3		3	3	3				1		3	3	12
$V_1=$	0			1	-1	-1	-1						-1	-1	-2
$V_2=$	1														1
$W_1^1=$	$\frac{1}{2}$	$\frac{1}{2}$	-1	$\frac{1}{2}$	$\frac{1}{2}$		$\frac{1}{2}$	$-\frac{1}{2}$							$\frac{1}{2}$
$W_1^2=$	$-\frac{1}{2}$	$-\frac{1}{2}$	1	2	$-\frac{5}{2}$	$-\frac{5}{2}$	-3		$\frac{1}{2}$				$-\frac{5}{2}$	$-\frac{5}{2}$	$(-\frac{11}{2})$
$W_2^1=$	1		1		-1		-1			$-\frac{1}{3}$	$-\frac{1}{3}$		-1	-1	-2
$W_2^2=$	1		-1		1		1						1	1	4
$W_3=$	2			$-\frac{5}{2}$	2	2	$\frac{5}{2}$					$-\frac{1}{2}$	2	2	2
$y_{11}=$	1			-1	1	1	1						1	1	2
$y_{13}=$	0			$\frac{5}{4}$	-1	-1	-1						-1	-1	-1
$X_{32}=$	3	1	1	-4	2	2	3				-1		2	1	-1
$X_{12}=$	-1	-1	2	4	-5	-5	-6				-2		-5	-5	-11
$V_3=$	1			$-\frac{5}{4}$	1	1	1						1	1	1
$X_{11}=$	1	1	-2	1	1	1	1								1

	E_1	E_2	E_3	a_i
A_1	1	-1	5	5
A_2	3	3		6
A_3	1	3		4
e_j	5	5	5	15

7		S_2	S_4	S_3	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	y_9	y_{10}	W_1^2
$-k=$	$-8627\frac{3}{11}$	$32\frac{8}{11}$	$84\frac{6}{11}$	$167\frac{1}{11}$	$73\frac{7}{11}$	$133\frac{7}{11}$	$46\frac{4}{11}$	500	$46\frac{8}{11}$	440	390	500	$673\frac{7}{11}$	$623\frac{7}{11}$	$74\frac{6}{11}$
$X_{31}=$	$\frac{6}{11}$	$-\frac{16}{11}$	$-\frac{1}{11}$	$\frac{9}{11}$	$-\frac{3}{11}$	$-\frac{3}{11}$	$-\frac{8}{11}$		$-\frac{5}{11}$		1		$-\frac{3}{11}$	$\frac{8}{11}$	$\frac{10}{11}$
$X_{21}=$	$\frac{39}{11}$	$\frac{6}{11}$	$\frac{21}{11}$	$-\frac{24}{11}$	$-\frac{3}{11}$	$\frac{8}{11}$	$\frac{3}{11}$		$\frac{6}{11}$		-1		$-\frac{3}{11}$	$-\frac{3}{11}$	$-\frac{12}{11}$
$X_{22}=$	$\frac{21}{11}$	$-\frac{12}{11}$	$-\frac{9}{11}$	$\frac{48}{11}$	$-\frac{27}{11}$	$-\frac{27}{11}$	$-\frac{39}{11}$		$-\frac{12}{11}$		1		$-\frac{27}{11}$	$-\frac{27}{11}$	$\frac{24}{11}$
$V_1=$	$\frac{2}{11}$	$\frac{2}{11}$	$-\frac{4}{11}$	$\frac{3}{11}$	$-\frac{1}{11}$	$-\frac{1}{11}$	$\frac{1}{11}$		$\frac{2}{11}$				$-\frac{1}{11}$	$-\frac{1}{11}$	$-\frac{4}{11}$
$V_2=$	$\frac{10}{11}$	$-\frac{1}{11}$	$\frac{2}{11}$	$\frac{4}{11}$	$-\frac{5}{11}$	$-\frac{5}{11}$	$-\frac{6}{11}$		$-\frac{1}{11}$				$-\frac{5}{11}$	$-\frac{5}{11}$	$\frac{2}{11}$
$W_1^1=$	$\frac{5}{11}$	$\frac{5}{11}$	$-\frac{10}{11}$	$\frac{15}{22}$	$\frac{6}{22}$	$-\frac{5}{22}$	$\frac{5}{22}$	$-\frac{1}{2}$	$-\frac{1}{22}$				$-\frac{5}{22}$	$-\frac{5}{22}$	$\frac{1}{11}$
$y_{12}=$	$\frac{1}{11}$	$\frac{1}{11}$	$-\frac{2}{11}$	$-\frac{4}{11}$	$\frac{5}{11}$	$\frac{5}{11}$	$\frac{6}{11}$		$\frac{1}{11}$				$\frac{5}{11}$	$\frac{5}{11}$	$\frac{2}{11}$
$W_2^1=$	$\frac{13}{11}$	$\frac{2}{11}$	$\frac{7}{11}$	$-\frac{8}{11}$	$-\frac{1}{11}$	$\frac{10}{11}$	$\frac{1}{11}$		$\frac{2}{11}$	$-\frac{1}{3}$	$-\frac{1}{3}$		$-\frac{1}{11}$	$\frac{1}{11}$	$-\frac{4}{11}$
$W_2^2=$	$\frac{7}{11}$	$-\frac{4}{11}$	$\frac{3}{11}$	$\frac{16}{11}$	$-\frac{9}{11}$	$-\frac{20}{11}$	$-\frac{13}{11}$		$-\frac{4}{11}$				$-\frac{9}{11}$	$-\frac{9}{11}$	$\frac{8}{11}$
$W_3^1=$	$\frac{20}{11}$	$-\frac{2}{11}$	$\frac{4}{11}$	$-\frac{39}{22}$	$\frac{12}{11}$	$\frac{12}{11}$	$\frac{31}{22}$		$-\frac{2}{11}$			$-\frac{1}{2}$	$\frac{12}{11}$	$\frac{12}{11}$	$\frac{4}{11}$
$y_{11}=$	$\frac{9}{11}$	$-\frac{2}{11}$	$\frac{4}{11}$	$-\frac{3}{11}$	$\frac{1}{11}$	$\frac{1}{11}$	$-\frac{1}{11}$		$-\frac{2}{11}$				$\frac{1}{11}$	$\frac{1}{11}$	$\frac{4}{11}$
$y_{13}=$	$\frac{1}{11}$	$\frac{1}{11}$	$-\frac{2}{11}$	$\frac{39}{44}$	$\frac{6}{11}$	$-\frac{6}{11}$	$\frac{5}{11}$		$\frac{1}{11}$				$-\frac{6}{11}$	$-\frac{6}{11}$	$-\frac{2}{11}$
$X_{32}=$	$\frac{34}{11}$	$\frac{12}{11}$	$\frac{9}{11}$	$-\frac{48}{11}$	$\frac{27}{11}$	$\frac{27}{11}$	$\frac{39}{11}$		$\frac{1}{11}$		-1		$\frac{27}{11}$	$\frac{16}{11}$	$-\frac{2}{11}$
$X_{12}=$	0								1		-2				-2
$V_3=$	$\frac{10}{11}$	$-\frac{1}{11}$	$\frac{2}{11}$	$-\frac{39}{44}$	$\frac{6}{11}$	$\frac{6}{11}$	$\frac{5}{11}$		$-\frac{1}{11}$				$\frac{6}{11}$	$\frac{6}{11}$	$\frac{2}{11}$
$X_{11}=$	$\frac{10}{11}$	$\frac{10}{11}$	$-\frac{20}{11}$	$\frac{15}{11}$	$\frac{6}{11}$	$\frac{6}{11}$	$\frac{5}{11}$		$-\frac{1}{11}$				$-\frac{5}{11}$	$-\frac{5}{11}$	$\frac{2}{11}$
$S_5=$	$\frac{6}{11}$	$(-\frac{6}{11})$	$\frac{10}{11}$	$-\frac{9}{11}$	$\frac{8}{11}$	$-\frac{8}{11}$	$-\frac{3}{11}$		$-\frac{6}{11}$				$-\frac{8}{11}$	$-\frac{8}{11}$	$-\frac{10}{11}$

	E_1	E_2	E_3	a_i
A_1	$\frac{10}{11}$		$\frac{45}{11}$	5
A_2	$\frac{39}{11}$	$\frac{2}{11}$	$\frac{6}{11}$	6
A_3	$\frac{6}{11}$	$\frac{34}{11}$	$\frac{4}{11}$	4
e_j	5	5	5	15

8		s_5	s_4	s_3	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	y_9	y_{10}	W_1^2
$-K=$	-8660	60	30	120	30	90	30	500	430	440	390	500	530	580	20
$X_{31}=$	2	$-\frac{8}{3}$	$\frac{7}{3}$	3	$\frac{5}{3}$	$\frac{5}{3}$		1			1		$\frac{5}{3}$	$\frac{8}{3}$	$\frac{10}{3}$
$X_{21}=$	3	1	1	-3	-1						-1		-1	-1	-2
$X_{22}=$	3	-2	1	6	-1	-1	-3				1		-1	-1	4
$V_1=$	0	$\frac{1}{3}$	$-\frac{2}{3}$		$-\frac{1}{3}$	$-\frac{1}{3}$							$-\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{2}{3}$
$V_2=$	1	$-\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{2}$	$-\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{1}{2}$						$-\frac{1}{3}$	$-\frac{1}{3}$	$\frac{1}{3}$
$W_1^1=$	0	$\frac{5}{6}$	-1		$-\frac{1}{3}$	$-\frac{5}{6}$		$-\frac{1}{2}$	$-\frac{1}{2}$				$-\frac{5}{6}$	$-\frac{5}{6}$	$-\frac{4}{6}$
$y_{12}=$	0	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{2}$						$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$
$W_2^1=$	1	$\frac{1}{3}$	$\frac{1}{3}$	-1	$-\frac{1}{3}$	$\frac{2}{3}$				$-\frac{1}{3}$	$-\frac{1}{3}$		$-\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{2}{3}$
$W_2^2=$	1	$-\frac{2}{3}$	$\frac{1}{3}$	2	$-\frac{1}{3}$	$-\frac{4}{3}$	-1						$-\frac{1}{3}$	$-\frac{1}{3}$	$\frac{4}{3}$
$W_3=$	2	$-\frac{1}{3}$	$\frac{2}{3}$	$-\frac{3}{2}$	$\frac{4}{3}$	$\frac{4}{3}$	$\frac{3}{2}$					$-\frac{1}{2}$	$\frac{4}{3}$	$\frac{4}{3}$	$\frac{2}{3}$
$y_{11}=$	1	$-\frac{1}{3}$	$\frac{2}{3}$		$\frac{1}{3}$	$\frac{1}{3}$							$\frac{1}{3}$	$\frac{1}{3}$	$\frac{2}{3}$
$y_{13}=$	0	$\frac{1}{6}$	$-\frac{1}{3}$	$\frac{3}{4}$	$-\frac{2}{3}$	$-\frac{2}{3}$	$-\frac{1}{2}$						$-\frac{2}{3}$	$-\frac{2}{3}$	$-\frac{1}{3}$
$y_{32}=$	2	2	-1	-6	1	1	3		-1		-1		1		-2
$X_{12}=$	0								1		-2				-2
$V_3=$	1	$-\frac{1}{6}$	$\frac{1}{3}$	$-\frac{3}{4}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{1}{2}$						$\frac{2}{3}$	$\frac{2}{3}$	$\frac{1}{3}$
$X_{11}=$	0	$\frac{5}{3}$	$-\frac{10}{3}$		$-\frac{2}{3}$	$-\frac{2}{3}$			-1				$-\frac{5}{3}$	$-\frac{5}{3}$	$-\frac{4}{3}$

	E_1	E_2	E_3	a_i
A_1			5	5
A_2	3	3		6
A_3	2	2		4
e_j	5	5	5	15