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Production Planning Models And Linear Programming

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1- Introduction:

In the practical application of operations research plays the lenear programming methods the main roll in solwing many applied problems. This is hased on thier parctical simpel applications and also on thier simpel solution methods.

In economic field we can see that models can be formulated as linear programming problems due to liner structure and due to the non-negativity of the variables included in the Model.

The operations research process is considered from different byt intervelated perspectives.

From these per-pectives or components are: phasés, strategies and Factors. Each of these components consists of elements which are outlined. The relations between elements and between components are examined.

Even since the publication of the maximum principle the number of applications to economics problem has been steadly growing.

In this work we deal with the applicaltion of linear programming problem that can be formulated from production field.

Production fumctions form the basis of a preciese planning and control of costs.

In most accounting systems linear input output functions are supposed. Especially are presumed constant production coefficients and the possilsilty to allocate exactly at least variable costs to product units.

In this work it is analysed how for these presume correspond to modern production and cost theory. The influnce of multivariable and the ambiguous input-outp t relations in production processes and complex production structures to planning of costs is examissed.

The existence of several variables means that the differentiation aecording to production volume, normally used in cost paint of view. Therefore it is of on important to analysis the cost factor with it different types in order to achieve a more precise planning and control of costs.

In the following work we try to show how the input combination and different levels of costs affects productions planning.

At each section there is a mathematical linear programming model to allocates costs in it different types.

Also from the analysis of the characteristics of production conclusions are drawn for the planning of production processes taking in consideration the patterns of costs.

2- General describtion of production Models:

a- A linear programming problem is given as objective function under certain constraints, Model of linear programming are met in economic production planning. The traditional applied exampel is that economic Model of optimal production programm planning. In a production unit that produces X_j products, with m short term limited production factors V_i (i=1,2, ... m), (j=1, ... n) The production is given due to leontief production function.

Now if a_{oj} represents the profit per unit of production and let a_{ij} represent the input of the factor i to produce the product j. The problem is then formulated as follows.

The total profit will be as follows

$$\sum_{j=1}^{n} a_{oj} x_j = \text{..........} \max \qquad (1)$$

Under the constraints

$$\sum_{j=1}^{n} a_{ij} x_j \lesssim a_{io} \quad \text{For all } (i=1,2, \ldots m) \quad (2)$$

$$x_j \geqslant o$$

For this problem (1), (2), (3) there are many algorithms to get asolution of linear programming problems.

In applied field such problem is transformed to simple Integer programming, where the poducts x_j must be integer i.e the production sum x_j only integer is as (x_j=0 (mod1)) or in other formulation when the rest capicity must be integer.

The linear programming problem is transformed to

$$\sum_{j=1}^{n} a_{ij} x_j + \bar{x}_j = a_{,o} \quad (i=1,2, \ldots m) \quad (2')$$

by introducing the slack variabl $\bar{x}_i$ into (2) $\bar{x}_i$ represents the non-used capacity of the m factors V_i.

In case of integer value of the rest capacity there will be integer condition only for the variables $\bar{x}_j$

i.e $\bar{x}_j = 0$

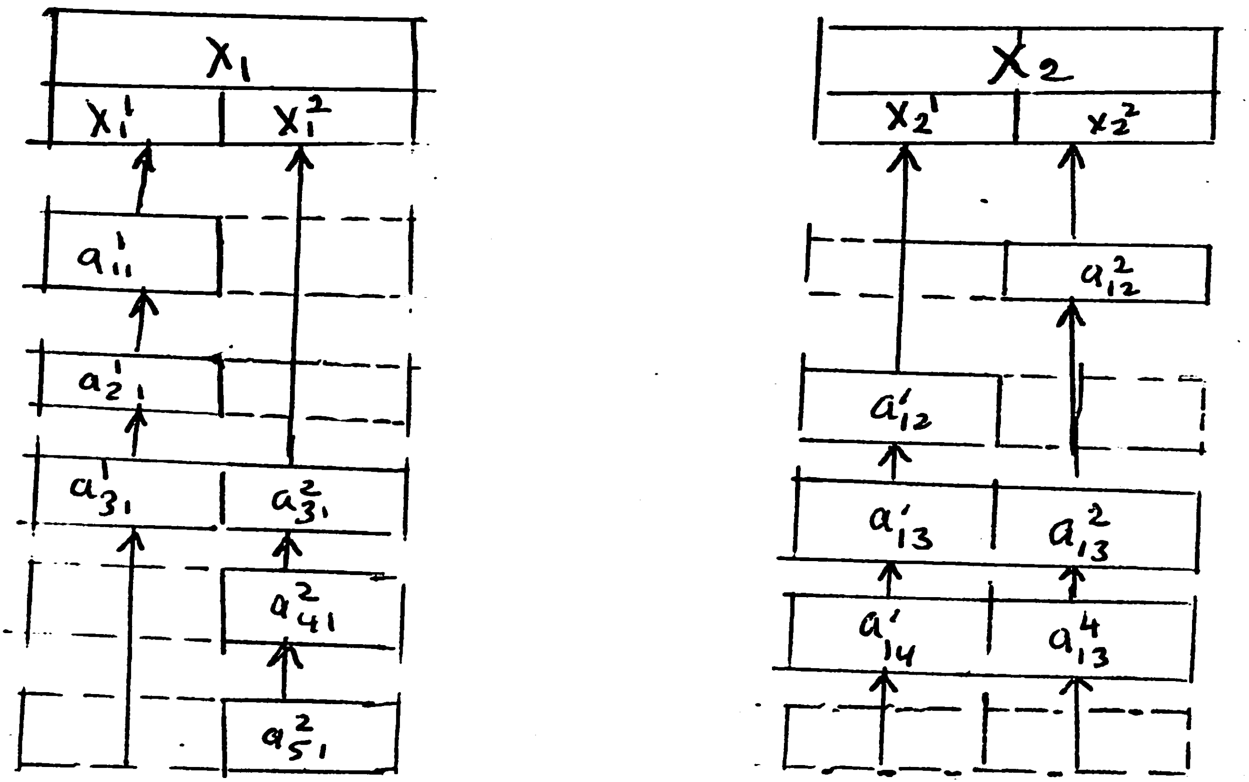
In the frist case (where the main variables are integers) The problem is called a real integer case. i.e for x_j with integer values of a_{ij} (i=1, ... m ; j=1,2, ... m) as also for x_i, but if for certain values of variables takes integer. Values then the problem is mixed - integer programming.

2-2 Production Planning model with (all) or (ether-or) decesion:

For production planning programm we mean heir that the production of one unit of product x_j (j=1,2, ... m) with the factors V_i, (i=1,2, ... m) and intensity a_{ij}.

Such Modeles of production have known and fixed production run. Such problems have itself solutions. The solution of these problems is given therough the answer of the question, how and with which combination of unites of products the production is produced.

A production unit is produced by different arts of mashines i.e for exampel any product as x_1 can be produced with V_1, V_2, V_3 or V_3, V_4, V_5 wile x_2 can be produced with V_2, V_3, V_4 or V_1, V_3, V_4, which means that for the product x_1, the factor product V_3 is neccessary while V_1 and V_2 can be replaced by V_4 and V_5 -, and for the product x_2, V_2 can be replaced by V_1



Fig(1)

Fig (1) shows that for x_1 there are two alternatives x_1^1 and x_2^2 auording to the process of production, V_1, V_2, V_3 or V_3, V_4, V_5 also for x_2 either through V_2, V_3, V_4 which produce V_2^1 or V_1, V_3, V_4 which produce x_2^2. The production factors needs production coeffients a_{ij}^k (k=1,2)

These production coefficient have the following properties

$$a_{31}^1 = a_{31}^2 \quad ; \quad a_{32}^1 = a_{32}^2 \quad ; \quad a_{41}^1 = a_{42}^2 .$$

Which helps in solving the problem through seperation of variables.

Since the production means are known then the problem can be formulated as follows

$$\sum_{i=1}^{n} a_{oj} \; x_j = \eta$$

or

$$\sum_{j,k} a_{oj}^{k} \; x_j^{k} = \eta \qquad (j=1,2, \ldots n;. \; k=1,2, \ldots p) \qquad (8)$$

(number of avialable substituation)

Under the constraints

$$\sum_{j,k} a_{ij}^{k} \; x_j^{k} \leqslant a_{io} \qquad (9)$$

$$x_j^{k} \geqslant o \qquad (10)$$

For the number of product x_j

$$\sum_{k} x_j^{k} = x_j \qquad (11)$$

Problem (8), (9), (10), (11) is the some as problem (1), (2), (3) but the last problem show that, the production of the product x_j can be through the process x_j^1 as also through the process x_j, .. x_j^p produced.

i.e x_j can be produced through different process at the same time.

The other case of production is to produce x_j by either x_j^1 or x_j^2 or ... x_j^p i.e in Fig (1) either the production is x_1^1 or x_1^2 both are the same in the some planning period

The constraints in this case will be as follows

$$x_j^r \geqslant 0 \qquad (j=1,2,\ \dots\ n,\ k=1,\ \dots,\ r-1,r+1,\dots p)$$

$$x_j^k \ = 0$$

this means that one variable must equal to zero. but in the case of p=2 we can write (*)

$$x_j^1 .\ x_j^2 \ = 0$$

in case of $p > 2$ we get the following set of constrants.

$$x_j^1 \ x_j^2,\ x_j^1 \ x_j^3,\ \dots\ x_j^1 \ x_j^{p-1},\ x_j^1 \ x_j^p \ = 0$$

$$x_j^2 \ x_j^3,\ \dots\dots \qquad x_j^2 \ x_j^p \ = 0$$

$$x_j^{p-2} \ x_j^{p-1},\ x_j^{p-2} \ x_j^p \ = 0 \qquad (12)$$

$$x_j^{p-1} \ x_j^p \ = 0$$

If for exampel x_j^1 0 then form the frist row of constraint in (12) we see that all other alternatives processer equal to zero.

The same is in case of $x_j^r > 0$ this gives that all processes x_j^{k+r} equal to zero (from the $r^{\underline{th}}$ row)

The non-linear coustraints of (12) which have the value 0 or 1 are row transformed to a linear form.

The frist constraint in (12) is

$$x_1^1 \; x_1^2 = 0 \qquad (12)$$

Now let us introduce as a new variable with the the values 0 or 1 then (12') can be written in the form

$$x_1^1 \leq M \delta \qquad (13a)$$

$$x_1^2 \leq M (1-\delta) \qquad (13b)$$

$$\delta \leq 1 \qquad (13c)$$

$$x_1^1, x_1^2, \delta \geq 0 \qquad (13d)$$

$$\delta = 0 \qquad (13e) \text{ modl}$$

i.e greater than or equal to zero (13d)

smaller than or equal to 1 (13c)

and integer (13e), it can take the value 0 or 1 only

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* See: Dantzig, G.B: On the significeuce of solving linear programing problem with sowe integer variables Econometrica 1960.

If δ equal to 1, that means x_1^1 equal to or smaller than M and x_1^2 must be smaller than or equal to zero (13b) and greater than or equal to zero (13d), which mean that it still for x_1^2 the value zero only.

In case of $\delta = 0$ then x_1^2 smaller or equal to M and x_1^1 can have the value zero.

The number M is a constant and it must be at least of great value, in order that x_1^1, x_1^2 not to be strong bounded.

For each of relation (12) can also have system of the following bounds as (13a) _____ (13e)

$$x_j^1 - M\delta_1 \leq 0$$

$$x_j^2 + M\delta_1 \leq M,\ x_j^3 + M\delta_1 \leq M;\ \ldots,\ x_j^{p-1} + M\delta_1 \leq M\ ;\ x_j^p + M\delta_1 \leq M$$

$$x_j^2 + M\delta_1 \leq 0$$

$$x_j^3 + M\delta_2 \leq M;\ \ldots\ x_j^{p-1} + M\delta_2 \leq M,\ x_j^p + M\delta_2 \leq M$$

$$x_j^{p-2} - M\delta_{p-2} \leq 0 \qquad (14)$$

$$x_j^{p-1} + M\delta_{p-2} \leq M,$$

$$x_j^{p-1} - M\delta_{p-1} \leq 0$$

$$x_j^p + M\delta_{p-1} \leq M$$

with

$$1 \geq \delta_k \geq 0 \qquad (15)$$

$$\delta = 0 \qquad (\text{Mod } 1) \qquad (16)$$

the relations (14) are written as

$$x_j^k - M\delta_k = 0 \qquad (k=1,2,\ ...,\ p-1)$$

(14')

$$x_j^{k+1} + M\delta_k \leq M \qquad (j=1,2,\ ...,\ P)$$

If we let for exampel in (14) x_j^2 greater than zero, then δ_2 must equal to 1, from the next row we find that

$$x_j^3;\ ...,\ x_j^p = 0$$

and from the relation of the row we see that for x_j^2 greater than zero, S_1 must be equal to zero

$$\text{jf } \delta_1 = 0 \quad \text{then} \quad x_j^1 = 0$$

The system (14) i.e (14') shows that for every j there is only one variable equal to zero.

This means that there are different ways of producing any product. So for exampel if we take 4 ways of production say x_1^1, x_1^2, x_1^3, x_1^4 for the production of x_1 and only two of this production ways are of maximum realiziation then we can get the following set of relations.

$$x_1^1 \quad x_1^2 \quad x_1^3 \quad = 0 \qquad (17a)$$

$$x_1^1 \quad x_1^3 \quad x_1^4 \quad = 0 \qquad (17b)$$

$$x_1^1 \quad x_1^2 \quad x_1^4 \quad = 0 \qquad (17c)$$

$$x_1^2 \quad x_1^3 \quad x_1^4 \quad = 0 \qquad (17d)$$

If two values of the variable x_1^i $(i=1,2,3,4)$ are equal to zero, means directly from (17a ... 17) that the values of the two other must equal to zero.

(17a) can be written as a linear relation as

$$x_1^1 \leq M\,(\delta_1 - \delta_2)$$

$$x_1^2 \leq M\,(1 - \delta 1)$$

$$x_1^3 \leq M\,(1 - \delta_2) \qquad (17a)$$

with

$$0 \leq \delta_1 \quad ; \; \delta_2 \leq 1 \qquad (18)$$

$$\delta_1, \delta_2 \equiv 0 \qquad (\text{mod } 1) \qquad (19)$$

If we use (17a) in (18) and (19) we get

δ_1	δ_2	x_1^1	x_1^2	x_1^3
0	0	0	0, >0	0, > 0
0	1	0, > 0	0, > 0	0
1	0	0, > 0	0	0, > 0
1	1	0, > 0	0	0

As it can be seen fron the above table we find that for values of δ_1 and δ_2 an i.e combiuation between δ_1 and δ_2 only two value for X_1^1 an till X_1^3 are obtained with values greater than zero system (17a) can be now written in a form suitable for the simplex method as hollows

$$
\begin{aligned}
x_1^1 - M\delta_1 - M\delta_2 &\leq 0 \\
x_1^2 + M\delta_1 &\leq M \\
x_1^3 + M\delta_2 &\leq M
\end{aligned}
$$

The value of M is choosen great enough such that the production sum is not strong bounded as the other bounds of of the linear programming problem, Anthor formulation of the system of constrants (12) in a linear programns is the following

$$
\begin{matrix}
x_j^1 & M\delta_1 \\
\vdots & \vdots \\
x_j^p & M\delta_p
\end{matrix}
\qquad (k=1, \dots p) \qquad (20)
$$

with

$$\sum_{k=1}^{p} \delta_k \leq 1 \qquad (21)$$

$$\delta_k \geq 0 \qquad (22)$$

$$\delta_{k=0} \qquad (23)$$

From (22) and (23) the variables $_k$ are integer and non-negative, Also from (12) only one value of the $_k$ takes the value one and all ather variables are equal to zero. The value that deduced from (20) of x_j shows that only one x_j) from x_j^k must be greater than zero.

Relation (21) shows that all processes x_j^k (k=1, ... p) must equal to zero.

If we cousider P processes x_j^k then relation (21) can be written as

$$\sum_{k=1}^{P} \delta_k \leq r \qquad (24)$$

For δ_k we must consider the infeger-non-negative condition, morover

$$\delta_k \leq 1 \qquad (28^-)$$

The two systems (20), (21), (22) and (23) or (20), (22), (23), (24), (25¯) can be considered as the developed system of alternative production processes, which can be produced with the relations (14'), (15), (16) and (17a), (18), (19).

The number of conditions is given through the application of system (20) to (15¯).

* See: Dinkelbach, . w and steffens, F.Gemischt ganzaahlige linear programms-zur losung gewisser Entscheidungs probleme. Unter nehmens foroching Bd 6. 1975.

III. Programm With Fixed Cost

1. The Problem

In production planning plays the fixed cost an important rolle for producing

X_j (j=1,2,...n). This fixed cost K_f affects not the lag of optimality but only its absolute value

$$G_{max} = G'_{max} - K_f$$

which gives

$$0 > G_{max} > - K_f$$

i.e. the production plan is always optimal.

For fixed costs there are three types .

1. Fixed cost that related direct to a product.
2. Fixed cost that related to a certain product interval (intervol fixed cost)
3. Fixed cost tnat related to department for the product (department cost).

2. Production Fixed Cost:

The objective function (cost function) by producing the products X_j with fixed cost K_j is given as follows.

$$K = \sum_j k_j X_j + \sum_j K_j \qquad \text{for } X_j > 0 \qquad (1)$$

$$K = o \qquad \text{for } X_j = 0 \qquad (2)$$

where K is the total cost, and K_j represents the unit cost for the variables X_j. K_j is the total fixed cost of producing X_j

Hiroh and Dantzig * introduce a new variable V_J in order to get solvable formulation of the problem where

$$V_j = \left\{ \begin{matrix} 0 \\ 1 \end{matrix} \right\} \qquad (3)$$

Using (3) in (1) and (2) we get the following relations.

$$K = \sum_j k_j X_j + \sum_j V_j X_j \qquad (4)$$

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* Hiroh W.M and Dantzig, G.B. The fixed change problem, Rand paper p-648 clalif. 1960.

where

$$V_j = 0 \quad \text{when} \quad X_j < 0$$

$$V_j = 1 \quad " \quad X_j > 0$$

which fulfill the condition

$$X_j \leqslant MV_j \qquad (5)$$

where M is a large number representing the upper bound of X_j

IF X_j = o then the value of V_j from(5) and (3) takes the values zero or one.

In the optimal solution V_j must have the value zero, since (4) takes its minimum and K_j must be minimized.

IF X_j 0 then V_j the value of V_j must equal to one.

Now let g_i be the porfit of unit of product X_j

Then we get the following function

$$\eta = \sum_j (g_j - k_j)\ X_j - \sum V_j k_j$$

to be maximizied under the constraints

$$\sum a_{ij}\ X_j \leqslant a_{io} \qquad (6)$$

$$X_j - MV_j \leqslant 0$$

$$X_j\ V_j \geqslant 0$$

V_j also can takes the value zero

where M the upper bound takes the form

$$M \geq \max_j \left(\min \frac{a_{io}}{a_{ij}} \right)$$

M must be definite.

IV. Interval Fixed Costs (Quantative Forms)

(Model a)

a. The first problem to be considered can be seen from the following Fig (2)

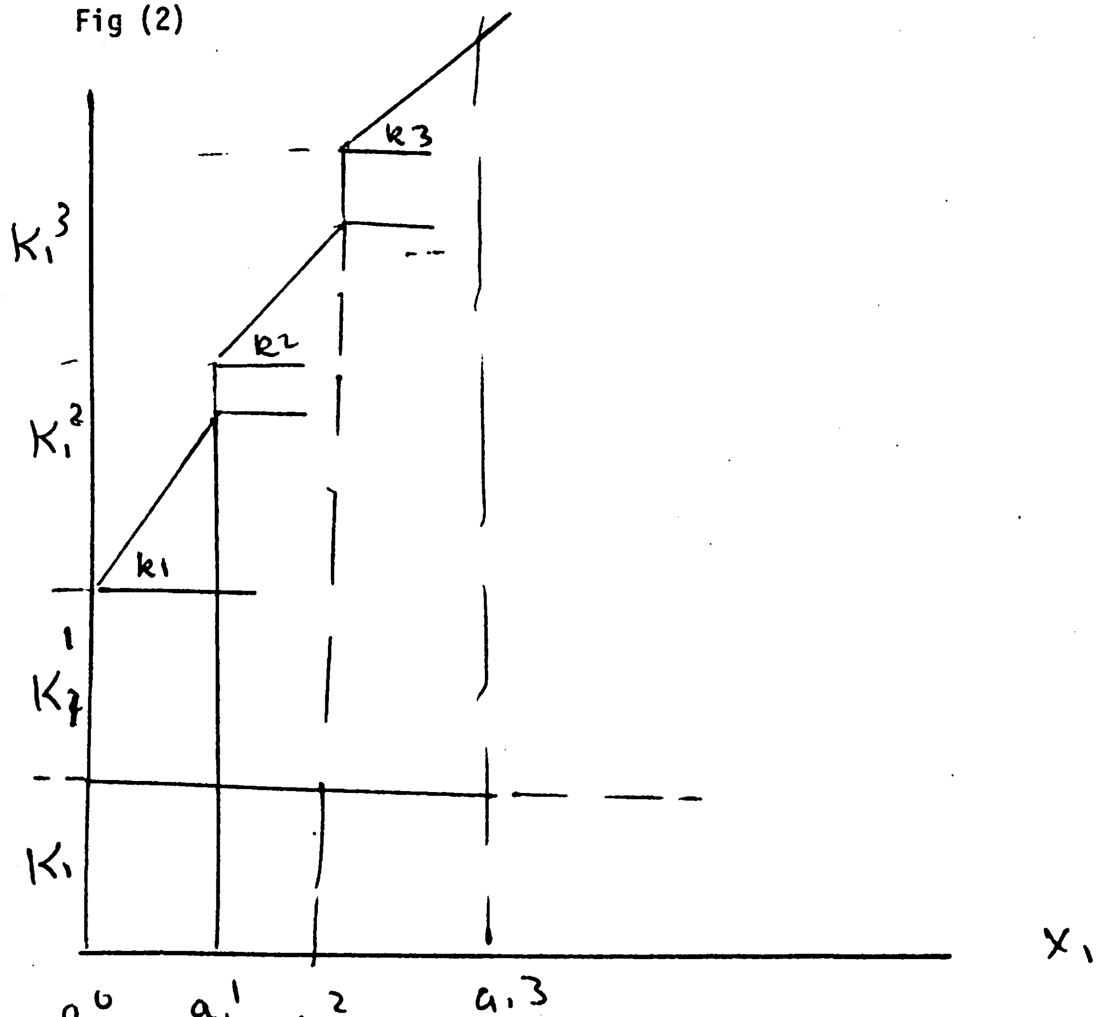

Fig (2)

In fig (2) we see that the total costs for producing product $X_1 \geq 0$ for exampel begins by:

i. the fixed production cost K_1

ii. the interval production cost a_i^i

iii. the interval fixed cost K_1^1

iv. the variable cost $k_1 X_1$

this is the same for the other products

The sequence of production must be taken in consideration i.e. 3 can not be produced before 2.

The same product can be produced by different method i.e. through different combinations of the casts as seen in fig (3)

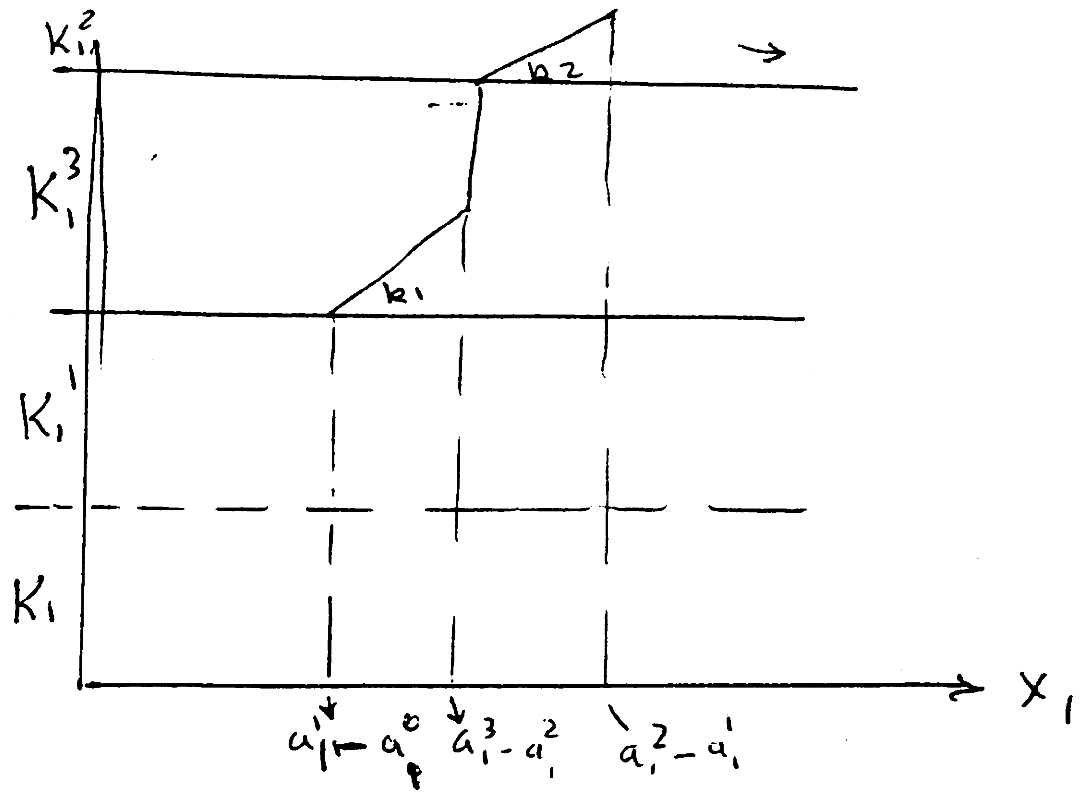

Fig (3)

Fig(2) and Fig(3) shows that some product can be produced with two ways of production. The capacity level determine the quantative level which is called capacity selection*. In dealing with interval fixed cast, we must differentiat between 3 models.

1. The production interval and the interval fixed costs are <u>equal</u>
2. The production interval and the interval fixed costs are <u>unequal</u> also the capacity selection is not allowed, that means, the sequence must be fixed.
3. The production interval and the interval fixed costs are unequal and the capacity selection is allowed.

In case models 1 and 2 the formulation of the problem as linear programming problem is simpel but in model 3 we must introduce a sequence constrants to the problem.

In case of one product we can solve the problem of interval fixed cost through cost comparsion, and still we have the case of more product.

See: Gutenberg. B.P.274.

In case of more product the linear programming formulation must shows the different factors on which the sum of products depends. In such case the intervals is not considered more than production intervals, since more products of the same character are produced.

Fig(2) and Fig(3) gives an exampel of using one production factor. This production factor shows that the aggregate 1,2 and 3 must be used and at the same time the intervals $a_1^1-a_1^0$, $a_1^2-a_1^1$, $a_1^3-a_1^2$ gives the production capacity of producing X_1 Product X_2 is also produced by using the factor 1. The intervals are to be measured by using this factor. If a_{io} is the capacity of the factor 1 and j=1,.....p are the aggregate with capacity a_{io}^1, a_{io}^2,.....,a_{io}^p then we have

$$\sum_{i=1}^{p} a_{1o}^{i} = \bar{a}_{1o}$$

In case of interval bounds we can write

$$(a_1^1-a_1^0) + (a_1^2-a_1^1) + (a_1^3-a_1^2) = a_{1o}$$

$$a_1^3 = a_1^0$$

with $a_1^0 = 0$

This seperation of the aggregates is used for all factors $i=1,\ldots,m$ which are used for the production of X_j, $(j=1,2,\ldots,n)$

b) <u>The Linear Programming Problem With interval fixed cost and Capacity Selection (Model b)</u>.

The model given in Fig(2) distinguish the objective function, that shows the sum of product X_1 with the proportional cost k_1X_1 also a real fixed cost with relation to the aggregates interval cost if X_1 represents the sum of product 1, then fig (2) and(3) gives the linear programming formulation as follows-for the costs

$$K = k_1x_1 + K_1 + K_1^1 + K_1^2$$

$$x_1 \leqslant (a_1^1 - a_1^0) + (a_1^2 - a_1^1)$$

$$x_1 \quad a_1^2$$

$$K = k_1x_1 + K_1 + k_1^2 + K_1^3$$

$$x_1 \leq (a_1^2 - a_1^1) + (a_1^3 - a_1^2)$$

$$K = k_1x_1 + K_1 + K_1^2 + K_1^3$$

$$x_1 \leqslant (a_1^2 - a_1^1) + (a_1^3 - a_1^2)$$

$$X_1 \lesssim (a_1^3 - a_1^1)$$

The first cost function is formulated from Fig (2) whill the second is obtqined from Fig(3). The last cost function is obtained since $K_1^2 < K_1^1$ For the product X_1 the following cost function is to be the main objective function.

$$K = k_r x_r + v_r k_r + \sum_{i=1}^{p} X_r^i K_r^i \qquad (7)$$

where

$$V_r = \begin{Bmatrix} o \\ or \\ 1 \end{Bmatrix}$$

also

$$x_r \leq MV_r \qquad (8)$$

$$x_r \leq \sum_{i=1}^{p} (a_r^i - a_r^{i-1}) \, w_r^i \qquad (9)$$

The above relations shows that for $x_r > o$ the variable v_r must take the value 1 and relation (9) allow for the variable w_r^1 to take the values zero or 1 according to the value of x_r, this can be deduced from the cost function (7)

IF we consider that the product X_j $(j=1,2,...n)$ can be produced with m factors $(i=1,2,....,m)$. These factors are in P aggregate part say (ℓ =1,...,) then the capacity of the production is given through the sum of parts of the capacity.

$$\sum_{\ell=1}^{p} a_{io}^{\ell} = a_{io} \quad \text{(for all } i=1,2,\ldots m)$$

and the production techniqual constraints are

$$\sum_{j} a_{ij} X_j \leqslant a_{io} \quad \text{(for all } i=1,\ldots,m)$$

under the system of constraints (7),(8) and (9) which gives

$$K = \sum_{j=1}^{n} k_j x_j + \sum_{j=1}^{n} v_j k_j + \sum_{i=1}^{m} \sum_{\ell=1}^{p} w_i^p \; (K_i) \qquad (7')$$

$$X_j \leqslant MV_j \qquad (8')$$

$$\sum_{j=1}^{n} a_{ij} x_j \leqslant \sum_{\ell=1}^{p} a_{io}^{\ell} \, w_i^{\ell} \qquad (9')$$

(for all i=1,2,....m)

The variables V_j, w_i^p takes only the values zero or one. The left hand side of relation (9) represents the total needed factors for the production of the sum X_j For production factors (that needed in production process must be smaller than the sum of total capacity for the aggregate parts.
For each w_i^ℓ (equal to one), we get the ability of production with known capacity a_{io}^ℓ. This shows that the objective function (7) includes only the interval cost (K_i).

As other case if $w^1 = o$ and $w^2 = 1$ the cost function (7') is to be minizied only and the aggregates to be consider are those with considerable intervol costs.

Since the total capacity with factors (i=1,2,....m) are equal to the some of total capacity, then (9') gives the boundes of capacity.

IF $w_i^\ell = k$, then the capacity of all factors i are totaly used and we have

$$\sum_{j=1}^{n} a_{ij} x_j \leq \sum_{\ell=1}^{p} a_{io}^\ell w_i^\ell = a_{io}$$

Now the final formulation of the above case as a quantative production with capacity selection will be the following linear programming problem.

$$\eta = \sum_{j=1}^{n} (g_i - k_j)\, X_j - \sum_{j=1}^{n} V_j K_j - \sum_{i=1}^{m} \sum_{\ell=1}^{p} w_i^{\ell} (K_i^{\ell}) \qquad (10)$$

$\ell \;\; i$ ....

is to be maximized, under the constraints

$$X_j - MV_j \leq 0 \qquad (11)$$

$$\sum_{j=1}^{n} a_{ij} X_j - \sum_{\ell=1}^{p} a_{io}^{\ell}\, w_i^{\ell} \leq 0 \qquad (12)$$

$$v_j ,\; w_j^{\ell} \leq 1 \qquad (13)$$

$$v_j ,\; w_i^{\ell} = 0 \qquad (14)$$

$$x_j,\; v_j,\; w_i^{\ell} \geq 0 \qquad (15)$$

where g_j is the unit profit of the product x_j and $(i=1,2,\ldots m)$

C- The Linear Program for Interval Cost Without Capacity Selection: (Model c)

In this model it is considered that the aggregate parts of factors of production must be in an integer sequence. The sequence is fulbilled due to a number of intervals. The sequence of the aggregate will be aggregate then aggregate 2 and so on.

In such case the programm 10 to 15 is written taking in consideration that

$$w_i^1 \geq w_i^2 \qquad (16a)$$

$$w_i^2 \geq w_1^3 \qquad (16b)$$

$$w_i^{p-1} \geq w_i^p \qquad (16c)$$

(for all $i=1,2,.....,m$)

That means w_1^3 can not take value = 1 and w_1^i, w_1^2=o, This means that the capacity of aggregate 3 in (12) must only be used when aggregate 1 and aggregate 2 are ready to work.

The problem in this case is that the objective function (10) is to be maximize under the constraints sequence (11) to (15) and moreover

$$\overline{w}_i^{\ell} \geq w_i^{\ell} \qquad \ell=1,2,....,p$$

d- The linear programm with the same interval and the same interval costs (Model d):

If the efficient intervals are of the some value, then we can say that the aggregate parts of the factors are the same. i.e. if we let the aggregate capacities as a_{i0} then

$$a_{i0}^{1} = a_{i0}^{2} = \ldots = a_{i0}^{p} = a_{i0}$$

In this case we did not care about the sequence or capacity selection.

Now let K_i represents the interval fixed cost, then we can put

$$K_i^1 = K_i^2 = \ldots = K_i^p = K_i$$

Now if we rewrite the system of relations 10 and (12), putting a_{i0}, K_i instead of w_i, Then the problem will be in the following form

$$\eta = \sum_{j=1}^{n} (g_j - K_j) X_j - \sum_{j=1}^{n} V_j K_j - \sum_{i=1}^{m} W_i K_i \qquad (17)$$

$$\sum_{j=1}^{n} a_{ij} X_j - a_{i0} W_i \leqslant 0 \qquad (18)$$

In relation (18) if we let w_i has the value > 1

If we put boundes to the variables w_i and letting it for certain number of intervals. i.e we take the factors that ready to be used. i.e. the capacity bounds are taken into consideration, we get the following linear programming problem.

$$\eta = \sum_{j=1}^{n} (g_j - k_j) X_j - \sum_{j=1}^{n} V_j K_j - \sum_{i=1}^{m} w_i K_i , \ldots \qquad (19)$$

(19) is to be maximized under the constraints.

$$X_j - M V_j \leqslant 0 \qquad (20)$$

$$\sum_{j=1}^{n} a_{ij} - a'_{i0} w_i \leqslant 0 \qquad (21)$$

$$(i = 1, \ldots m)$$

$$V_j \leqslant 1 \qquad (22)$$

$$V_j, w_j = 0 \qquad (23)$$

$$X_j, V_j, W_i \geqslant 0 \qquad (24)$$

with capacity boundes.

$$W_i \leqslant P_i \qquad (25)$$

$$(i = 1, 2, \ldots m)$$

P_i are constants and gives the number of efficient intervals (aggregate) for production factors i.

e- <u>Intervals fixed costs (more production case: (Model e)</u>.

In Model e the production interval sum is considered to be constant and independent of the given types of capacity of production.

The intervals fixed costs are given with respect to the different aggregate production factors. The production intervals are also constant.

This Model is different from that given by Model a and Model b, while in a and b the interval fixed costs are dependent on the optimal program Fig. (4) shows the total intervals for one product.

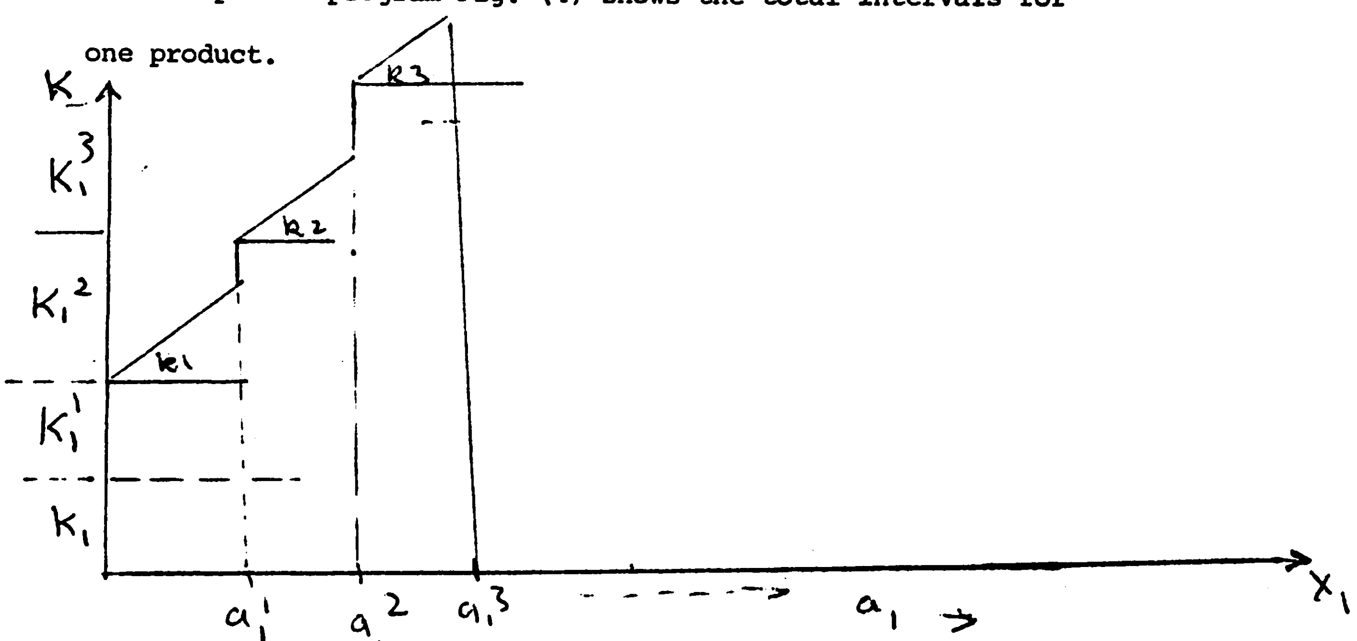

Fig. (4)

The interval bounds can not be taken as aggregate capacities but as a linear programm.

The total fixed costs for the production of X_1 is then not relatated to the sun K_1 this fixed costs with the constraints (3),(4) and 5 must be linear programm. beside the fixed cost K_1 playes the variable costs K_1 and also the interval fixed costs K_1^1, K_1^2 ... Then show the interval fixed costs the production intervals a_1^1, a_1^2,.

The costs run of Fig. (3) is written now through the costs functions

$$K = k_1 \ X_1 \qquad \text{when} \quad X_1 = 0$$

$$K = k_1 \ X_1 + K_1 + K_1^1 \quad \text{when} \quad 0 \leq X_1 \leq a_1^1$$

$$K = k_1 \ X_1 + K_1 + K_1^1 + K_1^2 \quad \text{when} \quad a_1^1 < X_1 < a_1 2$$

.

.

.

$$K = k_1 \ X_1 + K_1 + \sum_{\ell=1}^{p} K_1^{\ell} \quad \text{when } a_1^{p-1} \leq X \leq a_1^p$$

The number of production intervals is given with P.

The problem is now formulated as a linear programming problem in the form.

The objective function

$$K = \sum_{j=1}^{n} k_j X_j + \sum_{j=1}^{n} V_j K_j + \sum_{j=1}^{n} \sum_{\ell=1}^{p} W_j^{\ell} K_j^{\ell} \qquad (26)$$

with

$$\begin{array}{lll} V_j = 1 & \text{when} & X_j > 0 \\ V_j = 0 & \text{when} & X_j = 0 \\ W_j^{\ell} = 0 & \text{when} & X_j \leq a_j^e \\ W_j^{\ell} = 1 & \text{when} & X_j > a_j^e \end{array} \qquad (27)$$

If g_i represents the profit of unit of product of X_j, then the problem will be in the form.

$$\eta = \sum_j (g_j - k_j) X_j - \sum_j V_j K_j - \sum_j \sum_{\ell} W_j^{\ell} K_j^{\ell} \qquad (28)$$

is to be maximized under the constraints

$$\sum_j a_{ij} X_j \leq a_{i0} \qquad (29)$$

$$X_j - M V_j \leq 0 \qquad (30)$$

$$X_j - \sum_{\ell=1}^{p} (a_j^{\ell} - a_j^{\ell}) W_j^{\ell} \leq 0 \quad (a_j^0 = 0) \qquad (31)$$

$$X_j,\ V_j,\ W_j^{\ell} \geq 0 \qquad (32)$$

$$V_j,\ W_j^{\ell} \leq 1 \qquad (33)$$

$$V_j,\ W_j^{\ell} = 0 \quad (\text{mod } 1) \qquad (34)$$

and quese of the production is due to the condition

$$W_j^{\ell-1} > W_j^{\ell} \qquad (e = 2, 3, \ldots Q)$$

It can be seen from (30) in relation to (32), (33) and (34) that the interval fixed cost is taken in the objective function Also from (32), (33) and (34) the variable W_j^{ℓ} takes only the value zero or one for example the variables W_1^1, W_1^2, W_1^3 equal to one by the product sun of X_1 with the interval $a_1^2 < X < a_1^3$. In this case relation (31) are

$$X_1 - a_1^1\ W_1^1 - (a_1^2 - a_1^1)\ W_1^2 - (a_1^3 - a_1^2)\ W_1^3 \leq 0$$

$$X_1 - a_1^1\ W_1^1 + a_1^1\ W_1^2 - a_1^2\ W_1^2 + a_1^2\ W_1^2 - a_1^2\ W_1^3$$

$$-\ a_1^2\ W_1^3 - a_1^3\ W_1^3 \leq 0$$

$$X_1 \leq a_1^3 \quad \text{when } W_1^1,\ W_1^2,\ W_1^3 = 1$$

with which the production in interval (3) feasible.

In the objective function (28) beside K_1 and k_1 X_1 only the interval fixed coats K_1^1, K_1^2 and K_1^3 are taken into consideration and is maximized when $W_1^4, \dots W_1^p$ are zero.

It is also simpel case if we consider the value of the interval fixed cost as (K_j*) for certain product i.e.

$$K_r^1 = K_r^2 \dots = K_r^*$$

i.e.

$$a_r^1 = (a_r^2 - a_r^1) = \dots = (a_r^p - a_r^{p-1}) = a_r$$

and we get for the cost function 26 that

$$K = \sum_j k_j X_j + V_j K_j - \sum_j w_j K_j^*$$

with

V_j	$= 0$	when	$X_j = 0$
V_j	$= 1$	when	$X_j > 0$
W_j	$= 0$	when	$X_j = 0$
W_j	$= 1$	when	$0 < X_j < a_j$
W_j	$= 2$	when	$a_j < X_j < 2\, a_j$
$\vdots$			$\vdots$
W_j	$= P$	when	$(P-1)\, a_j < a_j < Pa_j$

from which we can see that the variables W_j is not now bounded of the value zero and one but can have the values from 1 to P., In this case the linear programming problem will be in the following form.

$$\eta = \sum_{j=1}^{n} (g_j - k_j) X_j - V_j K_j - \sum_{j=1}^{n} W_j k_j^{*} \qquad (2'8)$$

is to be maximized under the constraints

$$\sum_{j} a_{ij} X_j \quad a_{i0} \qquad (29')$$

$$X_j - M V_j \leq 0 \qquad (3'0)$$

$$X_j - W_j a_j \leq 0 \qquad (31')$$

$$X_j, V_j, W_j \geq 0 \qquad (32')$$

$$V_j \leq 1 \qquad (33')$$

$$V_j, W_j = 0 \quad \text{mod } X \qquad (34')$$

Re Relation (31') induces the integer values of W_j .. If there exist P Production intervals then W_j must be $\leq$ P and X_j has the bound P, a_j which is the capacity bound in the production Planning model.

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