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Unloaded Duplication System With Repair and Preventive Maintenance

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Introduction:

When we deal with the problem of finding the effect of combination of the preventive maintenance and repair to the life time of a two unit stand-by system. Two policies for inspection have been considered. Policy I, the operative unit undergoes inspection when the inspection time comes regardless of the state of the other unit, and policy II, the operative unit undergoes inspection when the inspection time comes, only of the other unit is n stand-by state. Thus no inspection takes places for the operative unit if the other unit is under repair or inspection.

[2],[3],[4], have studied this problem under the two policies, but in[1], [5] proved that there is no solution policy II, because the authers [3],[4], have follen in a number of faults.

In this paper we find a solution for the problem under policy II, and under the following assumption.

(i) The operational time of the first unit has a distribution function $F_1(x)$, and the second in the main state $F_2(x)$.

(ii) The repair time of the first unit has a distribution function $G_1(x)$, and for the second $G_2(x)$.

(iii) The time to inspection for the first equipment has a distribution function $U_1(x)$, and for the second $U_2(x)$.

(iv) The time from inspection to the end of inspection for the first unit has a distribution function $V_1(x)$, and for the second $V_2(x)$.

(v) The repair of the failed unit, or the inspection, starts immediately.

(vi) When the main unit breaks down or goes to inspection the reserval (if there is) instantly taken the load of the failed unit, or the unit under inspection.

(vii) The reapir and inspection return the unit completely to the normal state.

We use the following terminology.

$R(t)$ - denotes the probability that the duplication system will work smoothly to the instant t.

$R_1(t)$ - denotes the probability that the system in which the first unit failed at instant 0 will work smoothly to the instant t.

$R_2(t)$ - denotes the probability that the system in which the first unit is under inspection at instant 0 will work smoothly to instant t.

It is easily to see that $R(t)$, $R_1(t)$, $R_2(t)$ satisfy the following integral equation.

$$R(t) = \bar{F}_1(t)\,\bar{U}_1(t) + \int_0^t \bar{U}_1(x)R_1(t-x)dF_1(x) + \int_0^t \bar{F}_1(x)R_2(t-x)dU_1(x) \qquad \ldots \qquad (1)$$

$$R_1(t) = \bar{F}_2(t)\bar{U}_2(t) + \int_0^t \bar{U}_2(x)G_1(x)\bar{F}_1(t-x)dF_2(x) + \int_0^t \bar{F}_2(x)\;G_1(x)\;\bar{F}_1(t-x)dU_2(x) + \int_0^t \bar{F}_2(x)\;U_2(x)\;\bar{F}_1(t-x)\;\bar{U}_1(t-x)\;dG_1(x) +$$

$$+ \int_0^t \bar{U}_2(x)G_1(x)dF_2(x) \int_x^t \bar{F}_1(Z-x)G_2(Z-x)R_2(t-\tau)dU_1(Z) +$$

$$+ \int_0^t \bar{U}_2(x)G_1(x)dF_2(x) \int_x^t \bar{U}_1(Z-x)G_2(Z-x)R_1(t-Z)dF_1(Z-x) +$$

$$+ \int_0^t \overline{U}_2(x)G_1(x)dF_2(x) \int_x^t \overline{F}_1(Z-x)U_1(Z-x)R_2(t-Z)dG_2(Z-x) \quad +$$

$$+ \int_0^t \overline{F}_2(x)G_1(x)dU_2(x) \int_x^t \overline{F}_1(Z-x)V_2(Z-x)R_2(t-Z)dU_1(Z-x) \quad +$$

$$+ \int_0^t \overline{F}_2(x)G_1(x)dU_2(x) \int_x^t \overline{U}_1(Z-x)V_2(Z-x)R_1(t-Z)dF_1(Z-x) \quad +$$

$$+ \int_0^t \overline{F}_2(x)G_1(x)dU_2(x) \int_x^t \overline{F}_1(Z-x)U_1(Z-x)R_2(t-Z)dV_2(Z-x) \quad +$$

$$+ \int_0^t \overline{F}_2(x)U_2(x)dG_1(x) \int_x^t \overline{F}_1(Z-x)V_2(Z-x)R_2(t-Z)dU_1(Z-x) \quad +$$

$$+ \int_0^t \overline{F}_2(x)U_2(x)dG_1(x) \int_x^t U_1(Z-x)V_2(Z-x)R_1(t-Z)dF_1(Z-x) \quad +$$

$$+ \int_0^t \overline{F}_2(x)U_2(x)dG_1(x) \int_x^t \overline{F}_1(Z-x)U_1(Z-x)R_2(t-Z)dV_2(Z-x) \quad \ldots \quad (2)$$

$$R_2(t) = \overline{F}_2(t)\overline{U}_2(t) + \int_x^t \overline{U}_2(x)V_1(x)\overline{F}_1(t-x)\overline{U}_1(t-x)dF_2(x) \quad +$$

$$+ \int_0^t \overline{F}_2(x)V_1(x)\overline{F}_1(t-x)U_1(t-x)dU_2(x) + \int_0^t \overline{F}_2(x)U_2(x)\overline{F}_1(t-x)\overline{U}_1(t-x)dV_1(x) +$$

$$+ \int_0^t \bar{U}_2(x)V_1(x)dF_2(x) \int_x^t \bar{F}\,(Z-x)U_1(Z-x)R_2(t-Z)dG_2(Z-x)$$

$$+ \int_0^t \bar{U}_2(x)V_1(x)dF_2(x) \int_x^t \bar{F}_1(Z-x)G_2(Z-x)R_2(t-Z)dU_1(Z-x) \quad +$$

$$+ \int_0^t \bar{U}_2(x)V_1(x)dF_2(x) \int_x^t \bar{U}_1(Z-x)G_2(Z-x)R_1(t-Z)dF_1(Z-x) \quad +$$

$$+ \int_0^t \bar{F}_2(x)V_1(x)dU_2(x) \int_x^t \bar{F}_1(Z-x)V_2(Z-x)R_2(t-Z)dU_1(Z-x) \quad +$$

$$+ \int_0^t \bar{F}_2(x)V_1(x)dU_2(x) \int_x^t \bar{U}_1(Z-x)V_1(Z-x)R_1(t-Z)dF_1(Z-x) \quad +$$

$$+ \int_0^t \bar{F}_2(x)V_1(x)dU_1(x) \int_x^t \bar{F}_1(Z-x)U_1(Z-x)R_2(1-Z)dV_2(Z-x) \quad +$$

$$+ \int_0^t \bar{F}_2(x)U_2(x)dV_1(x) \int_x^t \bar{F}_1(Z-x)U_1(Z-x)R_2(t-Z)dV_2(Z-x) \quad +$$

$$+ \int_0^t \bar{F}_2(x)U_2(x)dV_1(x) \int_x^t \bar{F}_1(Z-x)V_2(Z-x)R_2(t-Z)dU_1(Z-x) \quad +$$

$$+ \int_0^t \bar{F}_2(x)U_2(x)dV_1(x) \int_x^t \bar{U}_1(Z-x)V_2(Z-X)R_1(t-Z)dF_1(Z-x) \quad \ldots (3)$$

Intbroduce the Laplace Transforms:

$$a_1(s) = \int_0^{\infty} e^{-st}\, \overline{F}_1(t)\, \overline{U}_1(t)dt \quad ,$$

$$a_2(s) = \int_0^{\infty} e^{-st}\, \overline{F}_2(t)\, U_2(t)dt.$$

$$b_1(s) = \int_0^{\infty} e^{-st}\, G_1(t)\, \overline{U}_2(t)dF_2(t) \; ,$$

$$b_2(s) = \int_0^{\infty} e^{-st}\, V_1(t)\, \overline{U}_2(t)dF_2(t).$$

$$c_1(s) = \int_0^{\infty} e^{-st}\, G_1(t)\, \overline{F}_2(t)dU_2(t) \; ,$$

$$c_2(s) = \int_0^{\infty} e^{-st}\, V_1(t)\, \overline{F}\,(t)dU_2(t).$$

$$\tilde{b}_1(s) = \int_0^{\infty} e^{-st}\, G_2(t)\, \overline{U}_1(t)dF_1(t) \; ,$$

$$\tilde{b}_2(s) = \int_0^{\infty} e^{-st}\, V_2(t)\, \overline{U}_1(t)dF_1(t).$$

$$\tilde{c}_1(s) = \int_0^{\infty} e^{-st}\, G_2(t)\, \overline{F}_1(t)dU_1(t) \; ,$$

$$\tilde{c}_2(s) = \int_0^{\infty} e^{-st} V_2(t)\, \overline{F}_1(t)dU_1(t)$$

$$e_1(s) = \int_0^\infty e^{-st}\, \bar{F}_2(t)\, U_2(t)\, dG_1(t) ,$$

$$e_2(s) = \int_0^\infty e^{-st}\, \bar{F}_2(t)\, U_2(t)\, dV_1(t).$$

$$\tilde{e}_1(s) = \int_0^\infty e^{-st}\, \bar{F}_1(t)\, U_1(t)\, dG_2(t) ,$$

$$\tilde{e}_2(s) = \int_0^\infty e^{-st}\, \bar{F}_1(t)\, U_1(t)\, dV_2(t).$$

$$d_1(s) = \int_0^\infty e^{-st}\, \bar{U}_1(t)\, dF_1(t),$$

$$d_2(s) = \int_0^\infty e^{-st}\, \bar{F}_1(t)\, dU_1(t).$$

$$R(s) = \int_0^\infty e^{-st}\, R(t)dt, \quad R_i(s) = \int_0^\infty e^{-st}\, R_i(t)\, dt, \; i = 1,2$$

In term of Laplace transform equation (1-3) will be after some rearrangment in the following form.

$$R(s) = a_1(s) + d_1(s)\, R_1(s) + d_2(s)\, R_2(s) \qquad \ldots \quad (4)$$

$$\left[1-b_1(s)\tilde{b}_1(s)-c_1(s)\tilde{b}_2(s)-e_1(s)\tilde{b}_2(s)\right]R_1(s)-\left[b_1(s)\hat{c}_1(s)+b_1(s)\tilde{e}_1(s)+\right.$$

$$\left.+c_1(s)\tilde{c}_2(s)+e_1(s)\tilde{b}_2(s)+e_1(s)\tilde{c}_2(s)+e_1(s)\tilde{e}_2(s)\right]R_2(s) =$$

$$= a_2(s) + a_1(s)\left[b_1(s) + c_1(s) + e_1(s)\right] \quad \ldots. \quad (5)$$

$$-\left[b_2(s)\hat{b}_1(s)+c_2(s)\tilde{b}_2(s)+e_2(s)\tilde{b}_2(s)\right]R_1(s)\left[+1-b_2(s)\hat{e}_1(s)-b_2(s)\tilde{c}_1(s)-\right.$$

$$\left.-c_2(s)\tilde{c}_2(s)-c_2(s)\hat{e}_2(s)-e_2(s)-\tilde{e}_2(s)e(s)-\tilde{e}_2(s)\tilde{c}_2(s)\right]R_2 =$$

$$= a_2(s)+a_1(s)\left[b_2(s)+c_2(s)+c_2(s)\right] \quad \ldots. \quad (6)$$

From equation (5), (6) we get

$$R_1(s) = \left[\left\{a_2(s)+a_1(s)(b_1(s)+e_1(s)+e_1(s)\right\}\left\{1-\tilde{b}_2(s)\tilde{e}_1(s)-c_2(s)\tilde{c}_2(s)+\right.\right.$$

$$\left.\left.+c_2(s)e_2(s)-e_2(s)e_2(s)-e_2(s)\tilde{c}_2(s)\right\}\right]+$$

$$+\left\{a_2(s)+a_1(s)(b_2(s)+c_2(s)+e_2(s)\right\}\left[b_1(s)c_1(s)+b_1(s)e_1(s)+\right.$$

$$\left.\left.+c_1(s)+c_2(s)+c_1(s)e_2(s)\right\}\right]+e_1c_2+e_1e_2\left.\right\}\Big] \Big/$$

$$\left[\left\{1-b_1(s)b_1(s)-c_1(s)b_2(s)-e_1(s)b_2(s)\right\}\left\{1-b_2(s)e_1(s)-b_2(s)c_1(s)-\right.\right.$$

$$\left.-c_2c_2(s)-c_2(s)e_2(s)-e_2(s)e_2(s)-e_2(s)c_2(s)\right\}-\left\{b_1(s)c_1(s)+b_1(s)e_1(s)+\right.$$

$$\left.+c_1(s)c_2(s)+c_1e_2(s)+e_1(s)c_2(s)+e_1(s)e_2(s)\right\}\left\{b_2(s)b_1(s)+c_2(s)b_2(s)+\right.$$

$$\left.\left.+e_2(s)b_2(s)\right\}\right]$$

$$R_2(s) = \Big\{1-b_1(s)b_1(s)-c_1(s)b_2(s)-e_1(s)b_2(s)\Big\}$$

$$\Big\{a_2(s)+a_1(s)\ b_2(s)+c_2(s)+e_2(s)\Big\} +$$

$$+\Big\{b_2(s)b_1(s)+c_2(s)b_2(s)+e_2(s)b_1(s)\Big\}\Big\{a_2(s)+a_1(s)(b_1(s)+c_1(s)+e_1(s)\Big\}/$$

$$/\Big\{1-b_1(s)b_1(s)-c_1(s)b_2(s)-e_1(s)b_2(s)\Big\}\Big\{1-b_2(s)e_1(s)-b_2(s)c_1(s)$$

$$-c_2(s)c_2(s)-c_2(s)e_2(s)-e_2(s)e_2(s)-e_2(s)-e_2(s)c_2(s)\Big\} -$$

$$-\Big\{b_1(s)c_1(s)+b_1(s)e_1(s) + c_1(s)c_2(s)+c_1(s)e_2(s)+e_1(s)c_2(s) +$$

$$+ e_1(s)e_2(s)\Big\}\Big\{b_2(s)b_1(s)+c_2(s)b_2(s)+e_2(s)b_2(s)\Big\}$$

Substituting these values $R_1(s)$, $R_2(s)$ in (4) we get $R(s)$.

The mean life time for the system is given by

$$T_2 = R(o) = a_1(o) + d_1(o)R_1(o) + d_2(o)R_2(o)$$

Now we consider the special case

$$F_1(t) = F_2(t) = F(t),\ G_1(t) = G_2(t) = G(t)$$

$$U_1(t) = U_2(t) = U(t)\ ,\ V_1(t) = V_2(t) = V(t)$$

We have

$$a_1(s) = a_2(s) = a \quad , \quad \tilde{b}_i(s) = b_i(s) \quad (i=1,2)$$

$$\tilde{e}_i(s) = e_i(s) \quad i = 1,2 \quad \tilde{c}_i(s) = c_i(s), \quad i = 1,2$$

and

$$R_1(s) = a(s) \frac{1-c_2(s) - e_2(s) + c_1(s) + e_1(s)}{(1-b_1(s))(1-c_2(s)-e_2(s))-b_2(s)(c_1(s)+e_1(s))}$$

$$R_2(s) = a(s) \frac{1-b_1(s) + b_2(s)}{(1-c_1(s))(1-c_2(s)-e_2(s))-b_2(s)(c_1(s)+e_1(s))}$$

and R(s) is given by the following formula

$$R(s)=a(s)\, 1+\frac{d_1(s)\left\{1-c_2(s)-e_2(s)+c_1(s)+e_1(s)\right\}+d_2(s)\left\{1-b_1(s)+b_2(s)\right\}}{(1-b_1(s))(1-c_2(s))-b_2(s)(c_1(s)(c_1(s)+e_1(s))}$$

and the mean life time T_2 is given by

$$T_2=a(o)\left[1+ \frac{d_1(o)(1-c_2-e_2+c_1+e_1)+ d_2(1-b_1+b_2)}{(1-b_1)(1-c_2-e_2) - b_2(c_1+e_1)}\right] \quad \ldots (8)$$

Formulas (7), (8) in given in [1]

Now, if T_1 is the mean life time of the duplication system under policy I and is given by [4]

$$T_1 = a \left[1 + \frac{d_1(1+c_1-c_2) + d_2(1+b_2-b_1)}{(1-b_1)(1-c_2) - b_2c_1} \right]$$

we can prove that T_2 is large than T_1 where T_2 is given by formula (8)

To prove this we have

$$T_2 - T_1 = a \frac{(1-b_1)\left[e_1(1-c_2)+e_2c_1\right] + b_2c_1(1-e_1)+b_2c_1e_2}{\left\{(1-b_1)(1-c_2)-b_2c_1\right\}\left\{(1-b_1)(1-c_2-e_2)-b_2(c_1+e_2)\right\}}$$

Now we consider the denominator .

Sin $V(t) > G(t)$, then

$$(1-b_1)(1-c_2-e_2)-b_2(c_1+e_1) > 1 - b_2+c_2+e_2 > 0$$

Sin

$$b_2+c_2+e_2 < \int_0^\infty V(t)\, dF(t) < 1$$

Also

$$\left[(1-b_1)(1-c_2)-b_2c_1\right] > (1-b_1)(1-c_2-c_2)-b_2(c_1+e_1) > 0$$

and consequently the denominator is positive.

Since each term in the numerator is positive. This prove that $T_2 - T_1 > 0$

Consider the special case

$$U(t) = \begin{cases} 1 & t \geq T \\ 0 & t < T \end{cases}$$

Then the mean life time of the system becomes

$$A(T)=a_T\left[1+\frac{\overline{1+F(T)}\left[\int_T^{\infty}(G(t)-V(t))dF(t)\right]+\overline{F}(T)\left[\int_0^T(V(t)-G(t))dF(t)\right]}{\left[1-\int_0^T G(t)dF(t)\right]\left[1-\int_T^{\infty}V(t)dF(t)\right]-\int_0^T V(t)dF(t)\int_T^{\infty}G(t)dF(t)}\right]$$

or in the form

$$A(T)=a_T\left[1+\frac{1-V_T(\infty)+G_T(\infty)}{\theta_1(1-V_T(\infty))+\theta_2 G_T(\infty)}\right] \qquad (1)$$

where $a_T = \int_0^T \overline{F}(t)dt = \int_0^{\infty}\overline{F}(t)dt - \int_T^{\infty}\overline{F}(t)dt = \lambda - C$

$$\theta_1 = \int_0^{\infty}\overline{G}(t)dF(t), \quad \theta_2 = \int_0^{\infty}\overline{V}(t)dF(t)$$

$$G_T = \int_T^t \overline{G}(t)dF(t), \quad V_T(t) = \int_T^t \overline{V}(t)dF(t)$$

In the above equation we use A(T) because the mean time is a function of T.

As the mean life time of the system with repair only is

$$A(\infty) = \lambda\left(1 + \frac{1}{\theta_1}\right) = \lambda \frac{\theta_1+1}{\theta_1} \qquad (2)$$

which can be derived by setting $T_0 = \infty$ in the equation for A(T)

Comparison of the mean times to the system failure

In this section, we dicus the optimum preventive maintenance policies using the results just obtained in the preceeding section. As a criterion of optimality, we apply the mean time to the first system failure we shall compare the mean time A(T) with the mean time $A(\infty)$ and derive the necessary and sufficient condition for the optimum policies exist.

Theorem:

The mean time to the first system failure A(T) is greater than $T(\infty)$ if and only if there exist a finite T^*, $0 \leq T^* < \infty$ such that

$$p(T^*) > \frac{\theta_1(1-V_{T^*}) + \theta_2\, G_{T^*}}{(\lambda-c)\, V_{T^*}}$$

where $$p(T^*) = \frac{1 - V_{T^*} + G_{T^*}}{\lambda + C\,\theta_1}$$

Proof

From (1) and (2) we have

$$A(T)-A(\infty)=(\lambda-c)\left[1+\frac{1-V_T(\infty)+G_T(\infty)}{\theta_1(1-V_T(\infty)+\theta_2 G_T(\infty))}\right]-\lambda\frac{\theta_1+1}{\theta_1} =$$

$$=(\lambda-c)\frac{1-V_T(\infty)+G_T(\infty)}{\theta_1(1-V_T(\infty))+\theta_2 G_T(\infty)}-\frac{\lambda+c\theta_1}{\theta_1} =$$

$$=(\lambda-c)\theta_1\frac{1-V_T(\infty)+G_T(\infty)}{\lambda+c\theta_1}\cdot\frac{\lambda+c\theta_1}{\theta_1\left[\theta_1(1-V_T(\infty)+\theta_2 G_T(\infty)\right]} -$$

$$-\frac{\lambda+c\theta_1}{\theta_1\left[\theta_1(1-V_T(\infty)+\theta_2 G_T(\infty)\right]}\cdot\left[\theta_1(1-V_T(\infty))+\theta_2 G_T(\infty)\right] =$$

$$=(\lambda-c)\theta_1 p(T)w(T)-w(T)\left[\theta_1(1-V_T(\infty))+\theta_2 G_T(\infty)\right]$$

$$=w(T)\left[(\lambda-c)\theta_1 P(T)-\left\{\theta_1(1-V_T(\infty))+\theta_2 G_T(\infty)\right\}\right] \quad (3)$$

where $$w(T)=\frac{\lambda+c\theta_1}{\theta_1\left[\theta_1(1-V_T(\infty))+\theta_2 G_T(\infty)\right]}$$

It is evident that $w_1(t_o)$ is positive for a finite T^*, $0 \leq T^* < \infty$. Thus only consider the bracket on the right hand side of (3). It is also evident that $P(T^*)$ is positive for all finite T^*. If there exist some finite to such that

$$p(T) > \frac{\theta_1(1-V_T(\infty)+\theta_2 G_T(\infty)}{(\lambda-c)\,\theta_1}$$

then, the right hand side of equation (3) is positive i.e. $A(T^*) > A(\infty)$ for such a T^*.

conversly, if there exist of finite T^* such that

$$A(T^*) - A(\infty) > 0, \text{ we have}$$

that

$$p(T^*) > \frac{\theta_1(1-V_{T^*}(\infty) + \theta_2 G_{T^*}(\infty)}{(\lambda-c)\,\theta_1}$$

Conclusion:

We conclude that it is better to use preventive maintenance under policy II, because the mean life time for the system under this policy is greater than the mean life time for the system under policy I, and the theorem shows that there does exist a point T^* satisifing a certain condition and improve that the mean life time for the system under preventive maintenance policy II is greater than the mean life time for the system with repair only.

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